Fair value accounting for unpaid losses:  
Response to the FASB discussion paper on insurance contracts

Introduction: Structure of this comment letter

Fair value rests on sound financial principles, which the FASB correctly formulated in SFAS 157, “Fair Value Measurements,” where it explained fair value accounting. These principles underlie modern financial theory and actuarial pricing of insurance contracts. Fair value may be an innovation for financial reporting, but it has been the foundation for financial and actuarial work for over a generation. The IASB also advocates fair value accounting for all assets and liabilities, and two Accounting Boards hope to converge.

New accounting systems do not please everyone. The change from traditional GAAP causes dissatisfied firms to send angry comment letters. The charges that the FASB and IASB do not understand insurance and that they have theoretical concepts that do not accord with insurance practice are particularly biting. In response, the IASB has abandoned fair value principles in favor of traditional practices that some critics favor. The FASB has gone even further, abandoning financial risk margins in favor of ad hoc amortization of profit margins.

Most criticisms of fair value proposals are specious. The FASB and IASB initially reasoned that the economic rationale for fair value accounting is clear. Insurance markets are competitive, so products are priced at their economic values and firms are valued at their economic values. This is correct, but the Boards lacked the actuarial and financial knowledge of product pricing and company valuation to explicitly justify fair value.

This comment letter outlines the principles of fair value. It explains how actuaries and financial economists derive fair premiums to price insurance contracts, and why many insurers fear fair value. It shows how to compute fair value risk margins for unpaid losses, both before and after claims are incurred.

Actuarial pricing and financial theory are not simple. The methods used by financial economists and actuaries to compute the fair value of unpaid losses involve capital requirements and the cost of holding that capital. This letter summarizes the issues for which the FASB must set accounting rules. The appendices explain how actuaries derive fair premiums from fair values of unpaid losses.

The FASB would be well served by a white paper setting forth the principles of fair value for unpaid losses and explaining how financial economists and actuaries derive fair values and fair premiums. Fair value properly portrays insurance operations; it should serve for external reporting the same role as for internal valuation. The FASB should base its accounting rules on established actuarial practice – not let its critics misleadingly distort fair value principles with obscure terminology.

The white paper should have underlying principles and examples of computing fair values. The principles are undisputed among practicing actuaries, but the application requires application of debated parameters. The white paper should explain why the cost of holding capital now ranges from 2% to 6%, depending on the country of domicile, investment yields, and the views of various financial economists. It should illustrate the common risk measures (value at risk, tail value at risk, expected policyholder deficit) and explain the advantages and drawbacks of each. But it should not dictate the valuation method. Financial theory evolves, changing valuation methods every decade. The white paper should reflect current financial theory, but it must realize that today’s truth may be tomorrow’s error.

Principle 1: Fair value is an attribute of the asset or liability, not of the firm holding the asset or liability.

Fair value reflects market value in active markets.¹ Active markets are competitive, so firms are price takers. The value of a good depends on its attributes, including the market supply and demand for the good, not on the attributes of particular firms. Insurance markets and investment markets are competitive: no investor or insurer has significant long-term market power.²
The fair value of traded assets does not depend on the party holding the asset. Consider an asset that pays 2,000 or nothing in 1 year with equal probabilities. Risk aversion depends on the wealth of the investor (among other characteristics). To a wealthy, risk-neutral investor, the asset is worth the present value of 1,000. To a poor, risk averse investor, the asset may be worth less than the present value of 1,000. But the asset has a single market value. Non-traded assets are similar. A receivable of 2,000 with a 50% probability of not being collected has an economic value. A firm with no material probability of ruin values the receivable at its probability adjusted cash flow, or 1,000. A firm in financial distress that would be bankrupt if the receivable is not collected may be willing to sell the receivable for less than its expected value. But the fair value is not the subjective value to the holder. If there is active market for the receivable, the market value is the fair value. For non-traded assets, pricing methods are best estimates of fair value. The implicit price of the receivable used to price products is its fair value. Firms price products assuming a probability that revenues are not received, and actuaries price insurance products based on economic values of losses, both paid and unpaid.

Some comment letters to the FASB say that the equivalence of fair value and market value is true only for traded assets in active markets. Insurance loss reserves are not traded, so their theoretical trading value is not relevant. Instead, the fair value is the certainty equivalent value to the insurer.

This argument is specious. Actuaries price the insurance contracts at their market values. They do not try to divine the certainty equivalent value of the loss reserves to the insurer, since this certainty equivalent value is irrelevant to market prices. Many assets – art work (paintings, sculptures), antiques, rare books – are worth high prices to wealthy collectors but little to the average person. The value of the object depends on what it can be sold for, not the value to the holder. The fair value of a liability is what others require to assume it, not the holder’s subjective valuation.

The notion that insurers value loss reserves and insurance contracts based on risk preferences that vary with their size or diversification is not reasonable. If the market price for an insurance contract is 10,000, a small, poorly-diversified insurer who writes the contract for 10,000 can not value the contract for external reporting at 9,000. If the market is competitive, the price is the same for all insurers who participate in the market.

In efficient markets, arbitrage arguments show that the market value of an asset is its value to diversified investors. Fair value accounting assumes the single price principle for all assets and liabilities when markets are competitive, even if they are not efficient. Investors do not deny arbitrage arguments for efficient markets, and financial economists do not dispute fair values in competitive markets. How ironic that the leading proponent of fair value accounting is forced by industry criticism to flee economic principles!

Fair value equals the price that a market participant would pay for the asset or liability in a competitive market with no transaction costs. Brokerage, underwriting, sales, and policy issue costs are not part of fair value. But benefits that are included in the insurance contract, such as defense costs, are liabilities just like pure losses.

The IASB has re-defined the fair value of loss reserves as the maximum amount the insurer rationally would pay to be relieved of the risk that the ultimate fulfillment cash flows exceed those expected. But economists, investment theorists, and actuaries agree that fair value does not depend on the party holding the asset or liability, and it surely does not depend on that party’s willingness to pay more or less for the asset or liability. The new definition is not even well-defined. An insurer has many parties who value loss reserves and decide how much to pay for them: managers, executives, and owners.

Underwriting managers are often compensated by bonus plans. A single large loss may eliminate the bonus, so managers often buy facultative reinsurance placements transferring large losses. The insurer’s CEO is not concerned with individual losses that are diversified in large portfolios. But CEO’s whose compensation depend on stable earnings buy catastrophe covers and aggregate excess-of-loss treaties (stop-loss treaties) and perhaps buy proportional treaties for surplus relief.

Shareholders (owners) diversify most efficiently by buying a portfolio of unrelated stocks. They want each firm to focus on its core competencies and take all risks with positive net present value. They are not afraid of
bankruptcy per se, unless it diminishes the a priori expected value of the firm. High cat covers protect franchise value and avoid some costs of bankruptcy; most reinsurance has a negative net present value.

**Principle 2: The fair value to the cash flow recipient is the fair value to the cash flow payer.**

A common mis-understanding is that recipients of risky cash flows should use a discount rate higher than the risk-free rate to compensate for the uncertainty in receiving cash and payers of uncertain cash flows should use a discount rate lower than the risk-free rate to offset the uncertainty in paying cash. Each person thereby values the risky cash flows at a certainty equivalent rate. The fair value of a risky cash inflow with a present value at risk-free rates of 10,000 is less than 10,000, since an investor may be willing to receive less to avoid the risks, and the fair value of a risky cash outflow with a present value at risk-free rates of 10,000 is more than 10,000, since an investor may be willing to pay more to avoid the risks. The probability distributions of cash inflows and outflows imply risk adjusted discount rates. The capitalization rate (discount rate) for the cash flow recipient is higher than the risk-free rate and the capitalization rate for the cash flow payer is lower than the risk-free rate.

This reasoning is emphatically rejected by modern financial theory and actuarial science. All cash flows have two parties: recipients and payers. Fair value does not depend on the perspective from which it is viewed. The asset of liability has a fair value whether or not it is traded. For traded assets, market values evident in actual transactions make the errors in the preceding paragraph seem childish. Non-traded assets and liabilities are no different, despite the lack of empirical data showing the errors. The FASB should not exchange economic principles for specious arguments.

Many assets and liabilities have three parties: payers, recipients, and taxing authorities. The issuer of a bond pays coupons; part is received by investors and part by taxing authorities. The fair value to the issuer = the fair value to the investor + the fair value to the tax authorities. The tax rate on an asset's cash flows affects the division of its fair value between investors and tax authorities. The same reasoning applies to insurance contracts. The fair value risk margin is paid part to tax authorities and part to other parties. The fair value risk margin is the total; ignoring the tax portion understates the risk margin by 60% to 70%.

Suppose the tax rate \( \tau \) is 35%, risk-free taxable bonds pay 10% coupons, and comparable tax exempt bonds pay 6.5% coupons. The bonds have different pre-tax cash flows but the same fair value. The tax effects on assets are undisputed; on insurance contracts, the tax effects are complex, and even good accountants err.

Tax rates vary by investor, but fair value depends on the marginal investor (the tax clientele), not the investor holding the bond. Tax exempt investors (charities, university endowments, non-profit organizations) receive higher after-tax income from taxable bonds, but the fair value of the bond does not change. Financial theory values assets based on marginal investors holding diversified portfolios, not on the value to the investor holding the asset, even if the investor receives cash different from that received by other investors.

Some insurers say the fair value of insurance liabilities depends on their liquidity. Illiquid assets appear to have lower market values than liquid assets with the same expected values. For example, private issue bonds that are not publicly traded have higher coupon rates than publicly traded bonds with the same default probabilities.

Modern finance has no theory for the relation of liquidity to market value. The anecdotal evidence is explained more simply by expected values. Investors may have to sell assets on short notice to meet cash needs. An illiquid asset sells for less, so it has a lower expected value. Liquidity affects expected values; it is not a separate attribute affecting fair value.

The illiquidity of loss reserves is used to justify fair values higher than discounted values at risk-free rates. The rationale is that insurer who wish to transfer loss reserves on short notice must pay more than their discounted value at a risk-free rate. But the justification for this view is dubious. Insurers rarely (if ever) have to transfer loss reserves on short notice, and the effect of liquidity on expected values is minuscule.
Liquidity suffers from the same perspective error as the investor’s risk aversion. To the cash flow recipient, illiquidity reduces the fair value; to the cash flow payer, illiquidity raises the fair value. But fair value is an attribute of the cash flow; unlike beauty, it is not in the eye of the beholder.

*Principle 3: Actuarial pricing of insurance contracts in competitive markets reflects their fair values.*

Fair value of loss reserves is the value of those unpaid losses used in pricing insurance contracts. This value depends on two items: required capital and cost of holding capital. The economic theory is straightforward: given the capital required and the cost of holding this capital, we derive fair value risk margins. But both items are hard to quantify, and reasonable estimates vary widely.

Value at risk, tail value at risk, expected policyholder deficit, RBC capital requirements, and rating agency capital standards are measures of required capital. The fair value of unpaid losses is used in pricing insurance contracts. These capital standards may be based on the VaR, TVaR, and EPD risk measures. Choosing one measure of required capital as the FASB or IASB standard is not meaningful. Required capital is what insurers believe they must hold, not what the FASB dictates.

Double taxation, agency problems, and incomplete financial disclosure affect the cost of holding capital. This cost varies by country: double taxation is high in the United States but low in some Eastern European nations; financial disclosure is strong in Western Europe but weak in some developing countries. The attached reports show how double taxation and financial friction costs affect fair premiums and the fair value of unpaid losses.

The role of the FASB is not to specify risk measures, costs of holding capital, financial theory, or ways to price insurance contracts. The fair value method for unpaid losses is their value in competitive markets, so the valuation method for unpaid losses is the valuation method to price insurance contracts. As valuation methods improve, estimates of fair value improve. The FASB specifies what to estimate; it may demand that actuarial methods be consistent with market prices and financial theory; it can not decree how actuaries derive the estimate.

A pricing method gives market prices, not theoretical prices that ostensibly represent *true* values that insurers *ought* to charge. Market prices are the final arbiter of fair value. A pricing method is biased if it routinely indicates rates above actual market prices. Insurers have methods to give any price desired; unless they use the method to price their contracts, they can not use the method to value unpaid losses. The pricing method should be consistent with — if not identical to — pricing methods used in other industries. The notion that insurance is unique and does not follow standard economic relations has no place in serious valuation work.

*The composite margin sacrifices fair value objectives for simplistic but erroneous proxies*

The FASB is concerned with accounting complexity and compliance costs. Fair values are the ideal; when active markets exist, they are simple and practically costless. The fair value of insurance loss reserves has been debated since the FASB’s first discussion memorandum in 1990. FASB’s composite margin purports to the complexity yet retain the substance of fair value accounting.

The fair value risk margin reflects required capital, costs of holding capital, and taxes on underwriting income. Quantifying the risk margin requires keen understanding of the insurer’s costs in holding reserves for unpaid losses. The required capital supporting unpaid losses and the costs of holding this capital are debated. No active market exists for unpaid losses, and financial economists disagree on the parameters of the valuation method. Long standing financial procedures are sometimes overturned by new studies. Accounting principles must consider cost-benefit tradeoffs: should risk margins be estimated even for unearned exposures or should composite margins take their place? Should risk margins be re-measured at each valuation date? Should they be re-measured when estimated losses change? If the risk margin or composite margin is amortized, what schedule should be followed? Does the risk margin or composite margin accrue interest, and at what rate?

Accounting should be simple and consistent; fair value risk margins are complex and may differ among firms. Simplicity has value, and the FASB has suggested a composite margin approach that eliminates the need to
derive risk margins. At inception of the insurance contract, the fair value of premiums equals the fair value of losses, expenses, and taxes, so the present value of premiums (at a risk-free rate) minus the present values of losses, expenses, and taxes equals the fair value risk margin. The FASB proposes to amortize this risk margin over the life of the claims based on a simple ratio of premiums and losses.

Simple is good; simplistic is dangerous. Re-valuing risk margins at each valuation date is an inefficient use of resources. But re-valuations are not needed: the justification for risk margins based on the cost of holding reserves for unpaid losses differs from the computation of the risk margins at each valuation date. Fair values are the implied market values, so risk margins must be consistent with market pricing of insurance contracts. But insurers do not re-measure the fair values of unpaid losses each accounting period from first principles. Actuaries are practitioners, not theoreticians; they work under business constraints, using cost-effective methods to value unpaid losses and price insurance contracts. Actuarial risk margins are loadings on the present value of unpaid losses or adjustments to the reserve discount rate. The loading / risk adjustment depends on the required capital for the block of business, the market interest rate, and the corporate income tax rate. Once the loading or risk adjustment is determined, it is used until the financial parameters change.

Casually unpaid losses generally decline by geometric decay, simplifying the risk margin computations. The loading (as a percentage of discounted losses) and the adjustment to the reserve valuation rate are relatively constant for all maturities. If the risk loading for a given line of business is 8% of discounted losses at 12 months of maturity, it is 8% (or close to 8%) at later maturities as well. Similarly, if the fair value of unpaid losses at 12 months of maturity is approximated by a 200 basis point risk adjustment to the reserve valuation rate, the same 200 basis point risk adjustment can be used at later maturities. The loadings and adjustments differ by line of business, sub-line, capital requirements, interest rate, and tax rate. For any block of business, they do not change materially as the losses mature.

If the FASB’s composite margin simplified insurance accounting with no loss of information, it would be ideal. But the composite margin replaces market values with biased formulas the prevent investors from properly valuing insurers. Proper risk margins are simple to apply and provide accurate information to users of financial statements.

The composite margin is marred by several errors, each detracting from its use to investors; together, they obviate the benefits of fair value accounting for unpaid losses. First, the composite margin eliminates fair value measurement for unearned exposures, when accurate estimates of insurer profitability is most needed. It smooths profits over the underwriting cycle, no less than the current unearned premium reserve and deferred policy acquisition cost asset, shedding no light on the expected profits of new business. Second, it dampens the net income effects of claim emergence, reducing profits of good experience and losses of poor experience. Third, the amortization schedule does not properly reflect the true run-off of the risk margin. Were the release of risk margins only a theoretical construct, one might excuse the error as an over-simplification. But the release of risk margins reflects cash payments to tax authorities and investors. These cash flows are easily modeled by simple formula; the erroneous amortization schedule serves only to obfuscate. Fourth, the FASB and the IASB whether the risk margin (or composite margin or residual margin) accrues interest. Their views suggest that accounting boards decide financial matters. The risk margin, being an after-tax item, accumulates at the after-tax risk-free rate, just as pre-tax losses accumulate at the pre-tax risk-free rate. Accounting edicts do not repeal economic laws.

The FASB agrees that the present value of unpaid losses must be re-estimated at each valuation date. This requirement is not disputed: if estimated unpaid losses increase, the reporting entity must disclose the change to investors. If fair value is a measure of market value, and the true risk margin increases when unpaid losses increase, the fair value risk margin should increase. Since insurers use loadings on discounted losses or risk adjusted discount rates, the effort is minimal. The FASB’s amortization of the composite margin is no simpler.

The fair value of unpaid losses should agree with actuarial pricing of insurance contracts. Valuing unpaid losses at CU 100,000 and pricing the business as if unpaid losses were CU 120,000 is inconsistent. But once
losses occur, insurers discard the a priori estimates and use actual information. Amortizing composite margins based on different estimates of unpaid losses discredits the process, turning fair values into ad hoc rules.

In perfectly competitive markets, the fair value of unpaid losses is the net premium minus expenses. But property-casualty insurance markets show strong underwriting cycles, with premiums fluctuating ±20% over their course. Insurers know the phase of the underwriting cycles as they price their policies, and the premiums charged may be well above or below the fair value of losses plus expenses. Fair value accounting is designed to show investors the true profitability of insurance operations. The composite margin eliminates insurers’ responsibility to disclose known and relevant information to investors, obviating the benefits of fair value accounting when they are needed most.

The triple taxation of investment income on capital and surplus funds (as investment income, as underwriting income on the policy’s profit, and as personal income to shareholders) is the primary source of fair value risk margins in the United States. The composite margin (in the hands of inexpert users) may imply that the fair value of unpaid losses subtracts anticipated tax liabilities along with other expenses. The tax liabilities are a cost of holding reserves; unless they are included in the risk margin, the fair value understates market value.

The FASB’s amortization schedule does not reflect the true amortization of the risk margin. Amortization schedules are useful if they are correct; the FASB’s is embarrassing. Why not treat insurance accountants and actuaries as professionals who can who can handle the correct amortization formulas? The objective of fair value accounting is to bring the value of unpaid losses in line with their market values. The risk margin is

$$\sum_{j=1}^{n} \frac{L_j \times K_j \times C_j \times (1 + r_f (1 - \tau)^j)}{(1 - \tau)}$$

$L_j$ is the unpaid losses, $K_j$ is the required capital supporting unpaid losses, $C_j$ is the cost of holding capital, $r_f$ is the pre-tax risk-free interest rate, $j$ is the accounting period, and $\tau$ is the corporate tax rate. Exact pricing considers several tangential items, such as how deferred tax assets affect required capital, how personal income taxes affect the cost of holding capital, and what capital supports unpaid losses. The amortization multiplies by the after-tax risk-free rate and subtracts the cost of holding capital in the previous period.

The FASB does not prescribe formulas. Over the past fifty years, financial research (Miller, Modigliani, Myers, Scholes, and many others) has improved our understanding of the cost of holding capital. The required capital for unpaid losses changes with regulatory directives (RBC, Solvency II) and with rating agency standards. As Best’s, Moody’s, Standard and Poor’s, and Fitch refine their models, insurers feel compelled to hold more or less capital for specific blocks of business. The FASB sets principles: fair value risk margins must accord with actuarial pricing of insurance contracts. The exact formulas change as pricing procedures improve.

The FASB and the IASB have a grade school squabble whether the residual margin or composite margin accretes interest. This quarrel decides nothing: fair value is market value, and accounting declarations don’t change economics. The risk margin formula above accretes interest at the after-tax risk-free rate, vindicating neither the IASB (since the losses accrue interest at the pre-tax risk-free rate) nor the FASB. Set fair value at market value and the arguments evaporate. Assets backing the risk margin earn investment income that is not offset by unwinding of the implicit interest discount in loss reserves. This is true regardless of the risk margin valuation method.

Public relations underlie insurers’ responses to FASB accounting principles

Recent comment letters to the FASB call out for explanation. If financial economists and actuaries agree that fair value does not depend on the risk aversion of the party holding the asset or liability, why did so many respondents to the FASB and IASB statements on fair value support the language in the exposure draft on
insurance contracts? Many respondents were actuaries working for insurers or consulting firms that employed actuaries and economists. The comment letter received by the FASB seem to belie the principles stated here.

Insurers fear they may suffer from fair value accounting. Insurers are disliked by many people. Some believe insurers earn excessive profits, despite the low returns in market data. This belief is shared by uneducated persons who view insurers as greedy corporations who cheat people and some well-educated persons who believe insurers contribute nothing to society. Populist politicians castigate insurers, promising to return their ill-gotten gains to consumers. Even respectable news-magazines carry articles exposing the sins of insurers.

This belief reflects the public’s relation with insurers. Consumers pay premiums at the beginning of the year. If all goes well and no claim is filed during the year, consumers receive no tangible good – except a bill for the renewal premium at the end of the year. If the insured has a loss and files a claim, the insurer investigates, subtracts a deductible, and may pay less than the insured had hoped for. If a claim is filed against the insured, the insurer provides a defense and settles the claim. The claim experience is unnerving, leaving consumers with unpleasant feelings.

Insurance consumers feel cheated, and they are cheated: by trial attorneys causing high costs for dubious claims, by dishonest persons filing fraudulent claims, by regulators who prohibit certain risk classes and raise rates for other insureds. Benefits received by consumers are less than half the premiums paid. The rest goes to plaintiff attorneys, defense counsel, agents, underwriters, and tax authorities. The benefits paid are often aggravated by dishonest lawyers, medical practitioners, and claimants; honest insureds required by law to buy insurance protection pay two to three times expected losses. The entire process is a drain on the productive economy. The fault lies not with insurers, who have always sought to combat fraud and improve efficiency. But fault is not relevant; blame is placed on the insurance industry.

Public perceptions adversely affect insurers several ways. Some legislatures and regulators restrict premium rates, class plans, and policy provisions. Class relativities by territory or sex are often prohibited, and overall premium rates are often suppressed, with the justification that insurers unfairly discriminate to earn excessive profits. The restrictions on insurance risk classification and policy pricing exacerbate the public disdain for insurers. If regulators prohibit personal auto classification by territory, urban drivers have trouble buying insurance in the voluntary market. They are covered in residual markets, where they receive poor service from insurers who don’t want their business. Sub-urban and rural drivers are overcharged; their expected benefits may be 30% to 40% of the premium. All consumers lose, and their anger is let loose on the insurance industry.

Some places use excess profits statutes to limit the income of insurers. The very notion of an excess profits statute reveals an antipathy towards insurers, who compete in free markets and generally earn below-average returns. Some excess profits statutes are particularly harmful. Insurance is cyclical: losses in bad years are offset by profits in good years. Excess profits statutes limit only profits, causing long-term losses for insurers.

Insurance profits intensity the calls to restrict the operations of insurers. Profits of other firms are viewed as testaments to sound business practice and innovations that improve consumer welfare. But profits of insurers are seen as stolen goods. In an insurer earns high profits, the public assumes it has cheated its consumers.

Under current GAAP and statutory accounting, insurers have meager (or negative) profits on new policy years even if the true profit is adequate. Insurers portray themselves as losing money each year on their operations or making only slight profits. The accounting presentation quells some of the dislike expressed by the public.

Some fair value proposals have inadequate risk margins, under-state the cost of unpaid losses, and expose insurers to charges of excess profits. These proposals are from analysts unfamiliar with product pricing in industries with material costs of holding capital and risk-based capital requirements. They mistakenly compute net present value and internal rate of return from cash flows of the insurer with its customers and suppliers. They leave out the high cost of holding capital in regulated industries and derive inadequate premiums and loss reserves. The over-stated income would aggravate public calls to reduce insurance rates and restrict insurers’ right to select or classify risks. The financial errors in these proposals are not easy to explain; even
good economists may err if they lack expertise in regulated industries. Insurance executive are not theoreticians; they can not explain the errors, but they know that the derived premiums are below market rates and the fair value loss reserves are below the economic values.

If the FASB mandates fair values below the economic values used by financial economists and actuaries, and its new accounting standards elicit public anger at insurers for excess profits (though true profits are below average), insurers will oppose the new accounting rules. The fault is not with fair value accounting: if unpaid losses are valued correctly, these values are the economic values in used pricing and company valuation. Insurers would show actual profits on their accounting statements, with no excess profits. The fault is not with insurers: if the FASB rules falsely arouse public ire against insurers, one expects strong opposition.

One wonders: “If insurers charge premium rates consistent with fair values of unpaid losses, why can’t they explain the pricing method to the FASB?” The question assumes insurers could not price policies without actuarial models. But the marvel of free markets is that economic theory explains human behavior, it does not cause behavior. Financial economists observe product prices and human behavior: premium rates adjust to clear markets – excess profits cause insurers to lower rates; inadequate profits lead them to raise rates.

Economic actors need not understand the pricing methods, even as animals need not read Darwin for evolutionary biology to work. After each advance of knowledge, we are mystified that earlier analysts misunderstood what now appears simple.

Markets force actuaries to price correctly; regulatory capture leads accountants astray.

Actuaries and accountants both seek the fair value of unpaid losses. One wonders: “Why should actuaries listen to actuaries? Perhaps actuaries should listen to accountants.”

Incentives differentiate actuaries from accountants. In competitive markets, actuaries have financial incentives to price products correctly. Over-pricing loses business; under-pricing loses money. Actuarial pricing starts with economic values of expected losses. If actuaries value an unpaid loss at 10,000 to price new policies in the same line of business, we presume the fair value of the unpaid loss is 10,000. In contrast, incentives for accountants setting rules are often perverse. Accountants are derided for not understanding industry practices even if the rules are justified: the history of fair value accounting provides ample examples. Economists speak of regulatory capture: regulation may be geared initially toward public welfare, but even idealistic regulators shift their views toward the industry perspective. The FASB deliberations on fair value illustrate how expert accountants stray from good accounting work in the face of strident criticism.

Actuarial pricing is complex; IASB members do not have time to learn how actuaries price insurance contracts. They hear about tail value at risk and return on risk adjusted capital and costs of holding capital. They include the actuarial terms in the accounting standards without understanding how the methods work. The FASB is more willing to learn the requisite pricing methods, but the complexity is still deterring.

Actuarial pricing methods used to set market premiums are consistent with modern financial theory and the pricing methods in other industries. The fair premium is the present value of benefits, expenses, and taxes. Pricing insurance contracts depends on accurately pricing the fair value of unpaid losses.

Actuarial pricing uses net present value and internal rate of return (discounted cash flow) methods, just as all business pricing uses. Insurance contracts are like other products and services: they are priced to an NPV of zero, an IRR equal to the opportunity cost of capital, an EVA (economic value added) of zero, and a RAROC (return on risk adjusted capital) equal to the opportunity cost of capital. These four pricing methods give the same indication; they are four expressions for the same financial relation.

In competitive markets, pricing produces returns appropriate for the risk of the project. The returns are after-tax and adjusted for inflation. If after-tax market returns are 4% in real (inflation-adjusted) currency units in competitive markets over the long-term, we presume the appropriate return is 4%.
Historical insurance returns may reflect low systematic risk or over-diversification.

Modern financial theory assumes the appropriate return varies with the risk of the project. It appears that new, rapidly expanding industries with high risk of failure have higher average returns, and mature industries with low risk of failure have lower returns. In efficient markets, arbitrage arguments indicate that expected returns depend on systematic (non-diversifiable) risk, not unique risk.

Insurers earn somewhat less than most other firms. The average risk premium (expected return minus risk-free rate) for insurers is about 90% of that for other firms. The expected return for insurers may vary by line of business, but empirical data are too sparse to derive reliable relativities. It appears that life insurance has lower expected returns than property-casualty insurance, but this relation also is hard to validate.

Financial economists give two possible causes for low insurance returns: low systematic risk and over-capitalized markets, each of which has good justification but different fair value implications. If the cause is low systematic risk, actuarial pricing should aim for a slightly lower than average return and the fair value of unpaid losses should use a risk adjusted discount rate higher than one might otherwise use. If the cause is over-capitalized markets, the low return reflects an excess of supply over demand for insurance. The low price of insurance contracts reflects an over-abundance of suppliers (insurers), not the economic value of the unpaid losses. The fair value of unpaid losses should be based on ideal markets.

Illustration: In an under-capitalized monopolistic market with a single insurer, prices are higher than they would be in a competitive market. Relating premiums to the present values of losses, expenses, and taxes would imply higher fair values for unpaid losses. In fact, the fair value of unpaid losses does not change, but the market power of the single insurer enables it to charge more than the fair value of premium. In an over-capitalized market with too many insurers, prices are lower than they would be in a competitive market. Relating premiums to the fair value of losses, expenses, and taxes would imply lower fair values for unpaid losses. But the fair value of unpaid losses does not change. The excess capital among insurers causes market premiums to be bid below their fair value.

Financial economists have tried estimating expected returns from risk measures without success. No financial theory of expected returns, whether Capital Asset Pricing Model or arbitrage pricing theory or behavioral finance, gives empirically validated estimates of future returns.

The risks of insurance operations quantified by value at risk, tail value at risk, and expected policyholder deficits are not systematic risk and have no theoretical relation to expected returns. Almost all underwriting risk, whether natural catastrophes or higher incidence of claims, is diversifiable by investors, and should not affect return on capital or economic value added. Insurers hold investment portfolios dominated by investment grade bonds, whose systematic risk is much debated by financial economists. Some analysts speak of debt betas; others interpret bond returns by expected defaults, market supply and demand for bonds of different maturities, and investor expectations of future inflation and interest rates.

This comment letter does not assume that insurance differs from other industry. It uses risk-free rates for fair values, with no risk adjustment. But the fair value risk margin is material, reflecting the cost of holding capital.

Discounted cash flow pricing methods reflect the fair value of assets and liabilities.

Discounted cash flow (NPV and IRR) pricing measure the return to suppliers of capital. Discounted cash flow pricing has two forms: the return on equity capital supplied by investors (owners) and the weighted average return on both debt and equity capital supplied by investors and creditors. The two forms give the same or similar prices. Debt issued by insurers is through holding companies or affiliates and does not appear on the insurer's financial statements. Most insurers do not carry debt, so we skip the complexities of weighted average cost of capital here. For simplicity, we use free cash flows to owners, or the return on equity capital.
NPV and IRR are equivalent ways to compute product prices. IRR is preferred by most business users, since it adjusts for the size and duration of the project. Most actuarial pricing (for both life and property-casualty insurance) uses IRR. The illustrations here use IRR (internal rate of return) on capital supplied by investors.

The relevant cash flows for NPV and IRR analyses are those between investors and the firm. Investors supply capital to firms, who use them to earn income which they remit to investors. We call these implied equity flows. Investors do not contribute capital or receive capital distributions each day. They contribute capital once every several years and receive stockholder dividends once a quarter. Moreover, firms have thousands of projects. An insurer sells tens of thousands of policies, each of which is priced by a discounted cash flow analysis. We model each policy as though investor contribute capital when the policy is written (capital supports reserves and RBC requirements) and receive back their capital as the claims are paid. In practice, the capital released as policies run off is used to support new business.

For firms with no reserves or risk-based capital requirements, implied equity flows to and from investors track the firm’s cash flows with consumers and suppliers. Cash received by the firm is equity of investors, and cash paid by the firm reduces investors’ equity. Non-cash flows that affect investors’ equity, such as changes in net working capital or required investment, are also modeled. Observed cash flows to and from the firm plus changes in net working capital and required investments are proxies for implied equity flows.

For banks and insurers, equity flows to and from investors are affected by reserve requirements, risk-based capital, and costs of holding capital, which are determined by statutory requirements, rating agencies, tax law, and financial friction costs. Academicians sometimes presume that economic values are not affected by statutes and tax law. But fair value is market value, which is affected by regulation, statute, and tax law.

Theoretical risk measures such as probability of ruin (value at risk), conditional tail expectation (tail value at risk), and expected policyholder deficit affect fair value through RBC requirements, rating agency capital standards, and managers’ desires for earnings stability. Value at risk is a percentile of the risk distribution; Tail value at risk and expected policyholder deficit are measures of the loss distribution. They have no intrinsic costs and are not themselves measures of value. They are combined with the cost of holding capital to set risk margins for unpaid losses.

Solvency II uses value at risk; Moody's and Fitch use tail value at risk; Best's uses EPD ratios. Specifying a risk measure for fair value risk margins is not meaningful. The measure affecting fair value is the one used to calibrate capital requirements. The risk measures change as actuaries, economists, rating agencies, and regulators develop better methods. Probability of ruin (= value at risk) was the first measure used by actuaries and investment analysts. It has been superceded by tail value at risk and expected policyholder deficit in many actuarial economic capital models. FASB telling economists and actuaries how to measure risk places the cart before the horse. The FASB’s role is to say: when reporting unpaid losses on financial statements, use the same values as for pricing insurance contracts in competitive markets.

*Statutory valuation of unpaid losses differs from statutory effects on loss valuation*

Some readers say: “Statutory and tax valuations of unpaid losses are for statutory and tax accounting. General accounting principles are not nation-specific. They depend on cash flows and other attributes of the losses, not of the country regulating insurance contracts.” This is correct. Statutory and tax valuations of unpaid losses are not relevant; the cash flows stemming from statutory and tax accounting affect the fair value of the losses.

We distinguish (a) statutory valuation from the effect of statutes on market value and (b) tax valuation from the effect of taxes on market value. Statutory accounting has an implicit risk margin: the interest discount in full value loss reserves. The risk margin in tax accounting varies by country; in the United States (the example in this report), it is zero. Neither the statutory risk margin nor the tax risk margin is the fair value risk margin, but statutory requirements and tax laws affect market values. The relations are clear for assets like tax exempt bonds (see below). Fair value measurement applies the same principles to liabilities.
Illustration: Investment income on tax exempt bonds (such as municipal bonds in the United States) is not part of taxable income. It has a lower tax value than investment income on taxable bonds, so the tax liability on tax exempt bonds is lower and the fair value of tax exempt bonds is higher. A tax exempt bond yielding r% per annum is worth more than a comparable taxable bond yielding r% per annum. The tax effects on unpaid losses are similar. If the tax value of unpaid losses is higher, the offset to taxable income is higher, the tax liability is lower, the insurer’s market value is higher, and the fair value of unpaid losses is lower. In the U.S. and most European countries, the tax value of unpaid losses is lower than the fair value, reducing the offset to taxable income and raising the tax liability, so the fair value increases.\(^6\)

The United States and some European countries require insurers to hold undiscounted (or partly discounted) reserves that exceed fair value, and they require additional capital for reserving risk, asset risks, and other risks that stem from holding unpaid losses. The cost of holding capital depends on financial friction costs. Most U.S. papers on the cost of holding capital consider double taxation, which is the easiest to quantify. The pricing illustrations included with this letter show how double taxation affects the fair value of unpaid losses.

International accounting standards are not nation-specific, so some say that tax law and insurance statutes should not affect fair value. But the use of nation-specific tax law is not a nation-specific accounting rule. The tax law and reserve requirements of the country in which the insurer is domiciled affect the market value of its unpaid losses. Accounting standards specify that market value is the best estimate of fair value. The items that affect market value are objective facts; they are not changed by accounting fiat.

Tax effects on asset yields and fair values were not always well understood. Tax laws are major determinants of fair value risk margins for unpaid losses, but they are so complex and range so much by country that some persons want to base fair value on pre-tax cash flows. Ignorance of tax law is no more an excuse for actuaries or accountants than it is for taxpayers.

Tax effects are better understood for assets than for liabilities, though the principles are the same. For traded assets, the tax effects are apparent in market values. We illustrate how tax law affects the fair value of assets and then extend the principles to liabilities.

The tax liability depends on the tax status of the investor. Tax exempt municipal bonds in the United States have large tax advantages for corporate taxpayers and wealthy individuals, smaller advantages for insurers (subject to proration) and less wealthy individuals, and no tax advantage for university endowments. This does not imply that the market values of the asset depends on the investor holding the asset, which would violate the principle of one price. Rather, assets and liabilities have tax clienteles which determine fair values. For many assets and liabilities, the tax clientele is a mix of investors. For example, stocks and bonds in the United States are held by both individual and corporate taxpayers. But individuals prefer stocks, corporations prefer bonds, and tax exempt investors hold no tax exempt bonds, reflecting their tax statuses.

Double taxation in the U.S. imposes an extra cost of \(\frac{\tau}{1 - \tau}\) on the investment income received by insurers on their capital funds. For a 35% corporate tax rate, the cost is half the investment yield. Other financial friction costs add 200 to 300 basis points, though the exact size is debated by economists. The cost of holding capital accords with the market premiums for insurance contracts.

Attachments: Actuarial pricing and valuation

This comment letter summarizes fair value principles for unpaid losses. The FASB can not rely on summaries to set accounting rules, lest it repeat the errors marring its exposure draft on insurance contracts. Wandering blindly through financial valuation of unpaid losses and actuarial pricing of insurance contracts invites the criticism that has plagued the FASB’s efforts to date. Setting standards without expertise in pricing and valuing insurance contracts is like setting standards for valuing financial options without knowledge of options pricing.

The attachments to this comment letter provide the actuarial pricing and valuation detail. The attachments are geared to readers with knowledge of insurance accounting but no previous experience with financial pricing.
Attachment A explains discounted cash flow pricing for insurance contracts, showing underwriting, investment, and tax cash flows and the resulting implied equity flows. The discounted cash flow model was first developed by economists at the National Council on Compensation Insurance for pricing workers’ compensation policies in the 1980’s, and the model is now used by the major U.S. property-casualty insurers. Mr Feldblum’s papers describing the model have been on the advanced ratemaking examination of the Casualty Actuarial Society since 1992. An earlier discounted cash flow model using net present values instead of internal rates of return was developed by Profs Stuart Myers and Richard Cohn for the Massachusetts rating bureau in 1982. Prof Myers was the first to relate the fair value risk margin to the cost of double taxation on the investment income on capital and surplus funds. An almost identical model for pricing life insurance and annuity products is described in the textbook by Atkinson and Dallas, written for the Society of Actuaries examination syllabus.

Attachment A uses graphics to highlight the implied equity flows that determine the net present value and the internal rate of return, and the leakage of insurer cash flows into tax liabilities that determine the fair value of unpaid losses. Attachment B is a simple three year model deriving the implied equity flows from underwriting cash flows, capital requirements, statutory reserves, and tax laws. These two attachments are abridged from a series of ten papers with complete documentation of discounted cash flow models for pricing insurance contracts published in the Casualty Actuarial Society Forum and the North American Actuarial Journal.

Attachment C explains how actuaries determine the fair value of unpaid losses. The procedures are the same as for pricing insurance contracts, since the fair premium equals the fair value of losses, expenses, and taxes. Most insurers use capital requirements from rating agency standards; large insurers also allocate capital from their internal ERM analyses, using value at risk, tail value at risk, or expected policyholder deficit measures.

The actuarial profession spent thirty years learning how to value unpaid losses, from the initial models by Prof Ferrari and by Prof Fairley in the 1970’s (both financial economists, not actuaries) to the RAROC (return on risk adjusted capital) models based on capital allocations from stochastic models. The FASB discussion paper would benefit greatly from proper understanding of the economic and actuarial bases of fair valuation. I have similar papers on risk measures, economic capital models, capital allocation, and return on risk adjusted capital, written to help actuaries understand target returns on capital and fair value of unpaid losses. In the past, the FASB has shown a desire to understand the economics of each industry so that the accounting rules properly reflect pricing. In contrast, the IASB seems to be rushing headlong and dragging the FASB with it.

The Excel spreadsheets show the computations of underwriting and investment cash flows, taxes, deferred tax assets, and implied equity flows for the simple three year illustration, as well as exhibits comparing the insurer’s cash flows with investors’ equity flows. The spreadsheets are heuristic: they are designed to clarify the actuarial calculations, not reflect actual cash flow distributions. Actual pricing studies for long-tailed lines of business generally use cash flow patterns for 15 to 20 years.

Endnotes:

1 Active markets are variously defined. Ideal active markets are competitive, liquid, efficient, and free of trading expenses. Firms in competitive markets face horizontal (perfectly elastic) demand curves. In efficient markets, past prices do not affect future prices; that is, prices are an autoregressive process of order 1. Insurance markets are sufficiently competitive that products are priced at their fair values.

2 In non-competitive markets, prices depend on the attributes of sellers and buyers (suppliers and consumers). In practice, lack of information by consumers and dominance of niche markets by some insurers may cause premiums to temporarily deviate from competitive prices, but actuarial pricing methods all assume competitive markets.

3 The IASB explains: “a risk adjustment should not represent the compensation a market participant would require for bearing the risk associated with the contract ... the objective of the measurement model is not current exit value or fair value and therefore does not reflect transfer to a market participant. Therefore, the risk adjustment should not be determined as the amount of compensation a market participant would require.”
The FASB quotes this new definition of fair value repeatedly, though it has not explicitly endorsed it.

Modern financial theory estimates fair value as the pre-tax equivalent yields; that is, the fair value is assumed to be fully taxable. Alternatively, cash flows can be recorded at their after-tax yields to the marginal investors, though this presentation is confusing when the reporting entity has a different tax status.

The IASB suggested three valuation methods for risk margins: value at risk, conditional tail expectation (tail value at risk), and cost of capital. VaR and TVaR are risk measures, not valuation methods. The cost of capital has several meanings: the opportunity cost of capital is a capitalization rate, not a valuation method. It appears that the IASB took three terms used by actuaries to lend sophistication to its exposure draft. The FASB cites the IASB methods without explicitly endorsing them.

In the United States, the tax value of unpaid losses is the discounted value at mid-term Treasury rates with no risk margin, which is lower than the fair value of the unpaid losses. The low offset to taxable income raises the tax liability and further reduces the fair value of the unpaid losses.
Mr Sholom Feldblum is CEO of New England Management Associates, which provides consulting services for insurers and other financial institutions. He has published 115 actuarial monographs, books, and papers, of which 17 are on the CAS examination syllabus, including eight monographs and papers on insurance accounting (GAAP, statutory, and fair value), three papers on tax accounting and tax strategy, a monograph on the NAIC risk-based capital requirements for property-casualty insurers, and papers on insurance pricing models for workers' compensation, personal auto, and commercial liability lines. He has received numerous awards from the actuarial societies for his contributions to actuarial science, and his papers are used in actuarial testimony, regulatory deliberations, and accounting standards.

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Attachment A: Financial Pricing Models for Property-Casualty Insurance Products

(White paper for the FASB deliberations on fair value of insurance liabilities)

SUMMARY

A financial pricing model derives the loss ratio needed to achieve a target return on capital from economic values of expected loss, expense, and tax cash flows. This report shows how implied capital flows, costs of holding capital, surplus requirements, target returns on capital, and investment yields determine target premiums. It focuses on the financial pricing concepts, which are valid across countries. Appendix A applies the pricing method to the accounting and tax laws of the United States.

Discounted cash flow models examine the net present value or the internal rate of return of investors’ cash flows. For most industries, the firm’s cash flows are reasonable proxies for the capital flows to and from the owners, if changes in net working capital and required investments are included. Cash inflows allow shareholder dividends, and cash outflows require capital contributions.

For insurers, statutory reserves and capital requirements limit the earnings that can be distributed to owners. Actuarial pricing of insurance products uses the implicit flow of funds to and from the suppliers of capital, or implied capital flows. Financial analysts use the term free cash flows; some actuaries speak of distributable earnings. The implied equity flows reflect the fair values of insurance liabilities: the economic values placed on them by investors.

The implied capital flows are determined by the company cash flows, statutory reserves, capital requirements, and tax laws. Cash flow projections are easier for insurance than for many other commercial products, since most underwriting costs are variable and demand for insurance is stable. This report assumes cash flows, reserves, and taxes have been estimated. It determines the implied capital flows used for net present values and internal rates of return and solves for the loss ratios or combined ratios that give a target return on capital.

This report uses graphics and basic illustrations to highlight fair value principles, with comparison of insurance pricing in different countries. The FASB accounting rules are not nation-specific, but each country has specific rules for loss reserves, required capital, taxes, and underwriting income. An appendix shows how double taxation in the United States, full value loss reserves, RBC requirements, and the definition of underwriting income determine fair premiums.
**Cash Flows and Capital Flows**

The opportunity cost of capital is the cost to capital providers, who can invest their funds in other projects of similar risk. The cost of capital depends on systematic (non-diversifiable) risk to capital providers, who can efficiently diversify unique risks, not the risk to the firm, which can not eliminate its business risk. The relevant cash flows are those between the firm and its investors (capital providers), which we call implied capital flows.

Most firms’ cash flows are proxies for capital flows. If a firm invests CU 1,000 at time 0 and receives CU 1,100 at time 1, we assume the firm’s owners provide CU 1,000 at time 0 and receive CU 1,100 at time 1. We use the firm’s cash flows to price its products.

In the figure below, the cash outflow for product development is the capital contribution at time t=0. The dividend to shareholders at later dates is the cash inflow from sales minus the cash outflows from production and further product development.

![Diagram of cash flows](image)

**Figure 1:** Capital contribution = firm’s cash outflow; firm’s cash inflow = capital distribution. After tax NPV of cash flows = NPV of capital flows

Profit or loss from accounting regulations, not related to cash flows, investment in new plants on equipment, or net working capital requirements, do not affect implied capital flows. Bankruptcy depends on a firm’s ability to meet debt obligations, not its statutory net worth. A firm can operate with negative equity, as long as it has the cash to pay its debts. A firm with positive equity that can not pay its debts may be declared bankrupt.
For insurers, the money that can be paid to capital providers is constrained by statutory reserves and capital requirements, in addition to the cash flows of the insurer. Statutory income minus the change in required capital is a proxy for implied capital flows.¹⁹

Capital requirements may depend on state RBC regulation, rating agency capital adequacy standards, ERM models, actuarial formulas, or market competition. For simplicity, we refer to risk-based capital requirements, without implying that the solvency requirements of the EU or the NAIC are the relevant constraints.¹⁰

In the figure below, the insurer receives premium from policyholders before paying expenses and claims. Capital flows and shareholder dividends reflect regulatory and market constraints on reserves and surplus.

<table>
<thead>
<tr>
<th>Financial Markets</th>
<th>Taxing Authority</th>
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<td>Insurer</td>
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<td>Policyholders &amp; Claimants</td>
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<tr>
<td></td>
<td>Regulatory and Market Restraints on Capital Flows and Dividends</td>
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<td>Capital Providers</td>
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**Figure 2:** The insurer's cash flows with policyholders, claimants, taxing authorities, and the financial markets, combined with regulatory restraints on capital flows and shareholder dividends, determine the capital flows to and from investors and creditors.

We compute the policy premium so that the IRR of the capital flows equals the opportunity cost of capital.

Statutory requirements reflect the promises to policyholders, so underwriting cash flows are inversely related to capital flows. Cash from policyholders must be held in reserves and further supported by statutory surplus.

- Premium collection, a cash inflow to the insurer, causes implied equity out-flows. Insurers pay acquisition and underwriting expenses at policy inception, but they must
hold gross unearned premium reserves in the U.S. (and some other countries) and capital supporting written premium risk charges.

- Loss payments, a cash outflow from the insurer, reduce loss reserves and the capital needed to support unpaid losses (such as the RBC reserving risk charge), allowing a return of capital to investors.\(^\text{11}\)

Accounting regulations of the country of domicile (not FASB rules) affect implied equity flows. The FASB allows acquisition expenses to be capitalized and amortized, with no surplus strain. If the insurer’s required capital is based on an accounting system (like U.S. statutory) with no deferred policy acquisition cost asset, investors must fund the initial underwriting loss (gross UEPR and full value loss reserves). The FASB rules determine how the implicit capital requirements inherent in state regulation affect the implied equity flows and the fair value of unpaid losses.

In the figure below, the insurer acts as a traffic controller. Premiums flow to UEPR and loss reserves, from which losses are paid. Capital is needed for statutory surplus and for equity in the policyholder reserves.

**Figure 3**: Capital contribution = reserves + surplus – premium; Capital distribution = reserves + surplus – losses. The premium supports fair value reserves (upper left arrow); investors and creditors contribute capital to support equity in reserves and surplus requirements.

Reserve and capital requirements are peculiar to each country. They depend on solvency regulation and rating agency standards.\(^\text{12}\) But pricing methods are universal. Insurance cash flows combine with regulation, capital constraints, and the costs of holding capital to determine the fair premium. The same pricing principles apply in tax havens with discounted reserves and low RBC requirements and in highly regulated countries with gross UEPR, full value loss reserves, capital constraints set by rating agencies and
regulators, and double taxation of shareholder dividends. The FASB’s valuation method should require that the fair value of unpaid losses be consistent with actuarial pricing of insurance contracts.

The figure below shows (i) uses of capital: implicit in statutory reserves and explicit in surplus requirements, and (ii) costs of capital: double taxation and financial friction costs (such as principal agent problems).

![Diagram of Reserve Requirements and Costs of Holding Capital]

**Figure 4:** Reserve requirements are high in the U.S. (gross UEPR and full value loss reserves) and lower in countries where insurers hold fair value reserves. Surplus requirements vary widely; in theory, they should be higher where reserve requirements are lower. Double taxation has been higher in the U.S. than in other countries. It was reduced in 2003, but it is still above average. Financial friction costs (principal-agent problems) are higher in emerging markets than in the U.S.

Some costs of holding capital are hard to quantify. Managers pursue their own interests, even if they conflict with those of investors. These financial friction costs, such as the costs of principal agent problems, are low in countries with strong protection for investors, such as the U.S. and western Europe. The firm’s managers serve the interests of shareholders, and government agencies (Securities and Exchange Commission in the U.S.) protect the interests of investors and bondholders. They are high in some emerging markets with less protection for investors. Published financial information is meager and owners have little recourse if managers are dishonest. These financial friction costs are added to the capitalization rate. Investors afraid that managers will use profits for personal gain, such as acquisitions of other firms at high market premiums, require a higher return on equity capital.
Regulation, tax law, and rating agency standards are the driving forces for required capital, costs of holding capital, and fair value risk margins. These items are not directly observed. The costs of holding capital reflect imperfections in capital markets, such as double taxation and principal agent problems. The investment yield on capital (after-tax) is subtracted from an assumed opportunity cost of capital to estimate the cost of holding capital. Required capital differs by valuation method: statutes give minimum amounts, and rating agencies give a variety of capital levels for each rating. Observed leverage ratios (premium to surplus; reserves to surplus) and market risk premiums in each country are the empirical data underlying the pricing methods.\[14\]

The figure below shows the influences on the capital needed by an insurer. Regulation, rating agencies, and market pressures differ by country and line of business. The capital needed and the cost of holding this capital determine the profit margin in the policy.

![Diagram](image)

**Figure 5** Regulators set minimum capital standards and reserves, but insurers hold much more than minimum levels. Rating agencies set target capital standards by rating level. Many insurers hold more than needed, reflecting market pressures for greater financial security. We infer market capital requirements by industry averages of premium to surplus ratios.

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Page 6  Financial Pricing Models for Property-Casualty Insurance Products
FREE CASH FLOWS AND IMPLIED CAPITAL FLOWS

Insurers are not conceptually different from non-financial firms. The practical distinctions reflect the relations of (i) inventories to insurance liabilities (reserves) and (ii) fixed assets to regulatory capital requirements.

- All firms use free cash flows for return on capital pricing models. For non-regulated firms, the firm’s cash flows minus required investment (capital expenditures) and the change in net working capital are proxies for implied capital flows.
- For insurers, only income in excess of minimum capital requirements may be distributed to investors. Statutory income is net cash flow minus the change in policyholder reserves. Net working capital is typically small and stable. Statutory income minus the change in required capital equals the implied capital flow. Insurers expected cash flows with consumers often have the opposite sign as equity flows.

Discounted cash flow pricing models treat policyholder reserves as inventories of cash, analogous to net working capital, and capital requirements like capital investments in fixed assets.

Illustration: A policy is written at time 0 for a $100,000 premium with acquisition expenses of $25,000. The investment yield is the risk-free rate of 7.95%. The insurer expects one loss at time 1 that will be settled for $110,000 at time 5. The discounted loss at time 0 is $110,000 / 1.0795 = $75,038. We compare the capital flows under U.S. statutory accounting vs fair value accounting with no risk margins, surplus, or taxes.

- At time 0, cash is debited $100,000 – $25,000 = $75,000, and unearned premium reserves are credited for $100,000. The cash inflow is $75,000 to the insurer, and the equity outflow is $25,000 from its owners.
- At time 1, investment income of $100,000 × 7.95% = $7,950 is received, the unearned premium reserve is taken down, incurred loss is debited for $110,000, and loss reserves are credited for $110,000. The cash inflow is $7,950, and the equity outflow is $10,000 – $7,950 = $2,050.
- Each subsequent year, the insurer receives investment income of $110,000 × 7.95% = $8,745 on the assets backing the loss reserves, which is an capital flow to investors.
- At time 5, loss reserves are debited by $110,000 and cash is credited for $110,000. The cash outflow is $110,000, and the capital flow from the loss payment is zero.

The implied capital flows are the basis for IRR or NPV calculations. From the investors’ perspective, the insurer purchases an unearned premium reserve for $100,000 at time 0, which is amortized over the policy year, and a loss reserve of $110,000 at time 1.

- The equity outflows are $25,000 at time 0 and $2,050 at time 1.
- The equity inflows are $8,745 at times 2, 3, 4, and 5.
For a fair value accounting system with no risk margins or capital requirements, the capital flows are zero.\textsuperscript{20}

\begin{itemize}
  \item At time 0, the net earned premium (after expenses) is $100,000 – $25,000 = $75,000 and cash is debited $75,000. The liability for unpaid losses on unearned exposures (the equivalent of unearned premium reserve) are credited $75,000 and incurred losses are debited $75,000. The cash inflow is $75,000 to the insurer, and the equity outflow is zero from the investors.
  \item At time 1, investment income is $75,000 \times 7.95\% = $5,963 and the liability for unpaid losses increases $5,693. The cash inflow is $5,693, and the capital flow is zero.\textsuperscript{21}
  \item Each year, the amortization of the reserve matches the investment income. The expected capital flow is zero each year.
\end{itemize}

The loss and premium reserves are inventories of money. The written premium paid by policyholders is not free cash that investors can use; it must be held by the insurer. Capital requirements (bonds and stocks held by insurers) are like capital investments (plant and equipment) of other firms.

\begin{itemize}
  \item The UEPR is a cash inventory: the net UEPR comes from the premium and the equity in the UEPR comes from investor. The written premium risk charge is a capital investment in bonds, not factories.
  \item The loss reserve is a cash inventory: the money comes from the net policy premium, supplemented (when the combined ratio is above 100\%) with investor funds. The reserving risk charge is a capital investment.
\end{itemize}

Country directives in the European Union (Solvency II) or North America (RBC) specify capital requirements. Fair value accounting uses the required capital, underwriting cash flows, investment yields, and tax law to derive fair value risk margin.

\begin{itemize}
  \item Loss and expense cash flows determine the bulk of the fair premium: about 90\% to 95\%, depending on the line of business.
  \item Statutory capital requirements and the treatment of acquisition expenses determine the initial underwriting loss and implied equity flow. Subsequent equity flows depend on the statutory valuation rate for loss reserves and the tax treatment of underwriting and investment income.
  \item The cost of holding capital and the implied equity flows determine the profit load in the insurance contract. Most profit (about 60\% in the U.S.) is collected by tax authorities; the remaining portion offsets the loss to investors from supporting insurance contracts.
\end{itemize}

IRR pricing focuses on implied equity flows. Implicit capital in full value loss reserves and gross unearned premium reserves is the same as explicit capital in surplus requirements. If U.S. regulators switch from gross to net (of acquisition expenses) unearned premium reserves and simultaneously require insurers to increase statutory surplus by the amount of these expenses, the capital flows and indicated premium do not change. Similarly, rating agencies do not care whether insurers hold capital as part of reserves or as surplus. If a
country changes from gross to discounted reserves, insurers may hold more surplus, with no change in total assets.

**CAPITAL FLOWS: THREE PERSPECTIVES**

We can determine the capital flows three equivalent ways.

- Total balance sheet (total assets) approach.
- Statutory income minus the change in surplus.
- Sum of implicit and explicit capital.

Each approach has its advantages, and is used for specific purposes.

- The total balance sheet (total asset) approach computes the IRR from underwriting flows, investment flows, and surplus requirements. It is used for iterative pricing models computing the indicated premium.\(^{22}\)
- Statutory income minus the change in surplus evaluates the IRR from book income and surplus requirements. This approach translates statutory income to economic income.\(^{23}\) It is most useful for accountants reconciling statutory income with economic income.
- The explicit plus implicit capital approach shows the sources of the capital flows. It shows why required capital is high or low in each scenario, and how required capital times the cost of holding capital gives the fair value risk margin. It is especially useful to explain the risk margin for unpaid losses.

1. Total assets approach:
   a. Compute the assets available from underwriting and investments =
      i. previous assets × investment yield + premium – expenses – paid losses – taxes
   b. Compute required assets = insurance liabilities (reserves) + required surplus.
   c. Available assets – required assets = capital distribution or capital contribution.\(^{24}\)

2. Statutory income minus the change in surplus requirements.
   a. Statutory income = earned premium + investment income – incurred loss – expenses ± direct charges and credits to surplus
   b. Subtract the change in the required surplus
   c. \(\Rightarrow\) Capital contribution (if negative) or capital distribution (if positive).\(^{25}\)

3. Explicit / implicit capital approach shows the intuition for the underwriting profit margin
   a. Implicit capital is the implicit interest discount in loss reserve + the equity in UEPR
   b. Total required capital is implicit capital + surplus requirements (explicit capital)
   c. The after-tax underwriting profit margin is the sum for all valuation periods of the required capital times the cost of holding capital
   d. The pre-tax profit margin is the after-tax margin / \((1 – \tau)\), where \(\tau = \) the tax rate

To reconcile the explicit / implicit capital with the total balance sheet approach:
Let economic income be fair value income with no risk margins
- Economic income at policy inception is a capital distribution
- Subsequent cash flows are funded by fair value reserves with no risk margins
- The change in explicit + implicit capital is a capital contribution or distribution
**Cash Inventories vs Free Cash**

Many non-financial firms hold fixed assets and inventories that are easily distinguished from free cash. Insurers hold inventories of cash. Distinguishing free cash flows from other forms of cash depends on required capital and constraints on cash dividends and debt repayment that may be imposed by regulation or statute.\(^{26}\)

A manufacturing firm holds inventories: finished goods, work in progress, and supplies. Net working capital includes inventories, receivables, store currency, pre-paid expenses (rent, group health insurance), and bank accounts. The change in net working capital is a use of cash; for manufacturers, most of this cash is used to buy or produce inventories.

*Illustration:* An auto manufacturer holds inventories of finished cars, semi-finished parts (chassis, seats, engines), and supplies of tires, glass, steel, upholstery, and so forth. Auto production requires much cash; the production cycle is several years long, with cash spent on designing, building, and testing prototypes and producing the cars. Investors’ cash is transformed into inventories that are locked up in the firm.

Most insurers hold little physical inventory. Production cycles vary:

Some insurers design their own products; others adapt policy forms from rating bureaus. Underwriting takes a few weeks, and some insurers bind policies immediately.\(^{27}\)

It seems that insurers do not need cash for net working capital. Whereas a manufacturer invests cash in net working capital before it produces its product, an insurer seems to create cash by its underwriting operations.\(^{28}\)

This reasoning is not correct. We contrast free cash with currency held by the firm. *Free* cash is not currency in cash registers or funds in checking accounts; it is the cash available to the owners (or capital providers) of the firm. The currency in cash registers is net working capital just like the items for sale on shelves; both are *uses of cash*. The currency in cash registers is an *inventory of cash*, but it is so small that it is not modeled explicitly.\(^{29}\)

When an insurer writes a policy, it uses the premium to buy an inventory of cash backing the unearned premium reserves.\(^{30}\)

- The manufacturer uses its owners’ cash to buy or produce inventories of supplies and semi-finished goods during the production cycle.
- The insurer uses cash available to its owners to buy inventories of cash (UEPR) while the premium is being earned.
A manufacturer’s inventory changes from raw materials and supplies to finished goods as it produces its products. Similarly, an insurer’s inventory of cash changes

- from unearned premium reserves before the exposures are earned
- to IBNR loss reserves before claims are reported
- to rough case reserves as the claims are investigated
- to accounts payable once the claims are settled before they are paid.

The owners and creditors of the manufacturer supply capital so that it can hold inventories over its product cycle. The inventories are shown as net working capital on the firm’s balance sheet, but the owners and creditors do not have access to this capital.

Similarly, capital providers supply an insurer with the cash to hold premium, IBNR, and case loss reserves. The cash held by the insurer is not longer available to owners.\textsuperscript{31}
COST OF HOLDING CAPITAL

One might say: “Inventories of goods and supplies are not fungible, have storage costs, and earn no income. But the assets backing insurance reserves have no storage costs, earn investment income, and can readily be sold to pay claims. What is the difference between cash available to owners vs an inventory of cash not available to owners?”

The difference stems from the cost of holding capital. If the after-tax investment yield on these reserves is less than the opportunity cost of capital (because of double taxation, agency costs, liquidity costs, or risk), holding cash or marketable securities is expensive. The assets earn investment income, but their real value declines each year.

The cost of holding capital applies to the funds contributed by capital providers, which depend on reserve and surplus requirements. We use the pre-paid acquisition expenses in the UEPR, the implicit interest discount in the loss reserves, the fair value risk margins, and the surplus requirements to assess the capital requirements.

Illustration: An insurer writes a policy for a $1,000 premium at time t=0 and pays agents’ commission of $200 and underwriting expenses of $100 at that time. The risk-free rate is 10% per annum, the expected loss ratio is 80%, and the average loss is paid two years after the inception of the policy.

In Country Y, insurers hold gross unearned premium reserves (the gross premium times the percentage of the policy term unexpired) and full value loss reserves.

- The net premium is $700.
- Investors add $300 at time t=0 to fund the unearned premium reserve.

In Country Z, insurers hold fair value liabilities for unpaid losses with a 20% risk margin (that is, 120% of the present value of the unpaid losses at the risk-free rate).

- The fair value liability at policy inception is $793.39.
- Investors add $93.39 at time t=0 to fund the unearned premium reserve.

The total required assets (reserves plus surplus) determines the capital flows, and required capital can not always be easily divided into reserves and surplus. If the insurance risks and rating agency evaluations are similar in the two countries, the capital flows should also be similar.

- A rating agency may expect more surplus from a well-reserved insurer than from a weakly reserved insurer for the same rating. An insurer that moves $100 million from loss reserves to surplus has not created $100 million of capital.
If rating agencies and capital markets are efficient, accounting entries are not relevant. The total assets held by the insurer determine its capital, not the division of assets between reserves and surplus.\textsuperscript{33}

Inventories of cash may be expensive. Transferring cash from the owner’s bank account to the inventory of cash (the premium and loss reserves) has several costs: tax costs from double taxation, agency costs from potential mis-use of the money by the insurer’s managers, and costs of risk and liquidity from investment and insurance risks.

\textit{Illustration: Double taxation}

The cost of double taxation varies by country: high in the U.S. in the 1970's, moderate in countries with low tax rates on equity income (current Western European and U.S.), and zero in some emerging economies in Eastern Europe.

- The investment income from the cash held in the owner’s bank account incurs one layer of tax: personal income taxes paid by the owner.
- The investment income from the cash held by the insurer incurs two layers of tax: corporate income taxes paid by the insurer and personal income taxes paid by the owner upon receipt of dividends from the insurer, as shown in the figure below.\textsuperscript{34}
Double taxation, agency costs, risk margins, and liquidity margins give the after-tax cost to the investors.

- Were this cost paid directly by policyholders to investors, it would be the full cost of holding capital.
  - But policyholders pay this cost to the insurer as a profit margin in the premium.
  - Insurers pay tax on this profit and remit the remainder to their investors.

The figure below shows this two-part cost of holding capital.

- First leakage: Capital needed to support insurance operations is costly. Policyholders reimburse investors this cost, so they receive a fair return on their investment.
Second leakage: This reimbursement is taxed as profit to the insurer. We gross up this cost to the pre-tax underwriting profit margin.\textsuperscript{35}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Leakage #1 is double taxation and other costs of holding capital (agency costs, risk, liquidity). If investors are to provide funds to support insurance operations, policyholders must reimburse these costs as additional premium. This reimbursement is taxed as underwriting profit, so the first leakage is grossed up for the pre-tax premium (second leakage).}
\end{figure}

If reserves are held at fair value with no risk margins and insurers hold no surplus, the premium pays expenses and claims. Investors contribute no capital and no money leaks out. The cash flows at the top of the figure below are also the capital flows. A zero cost of holding capital (by definition) has the same effect. Investors receive the appropriate return from investment of their capital.
Figure 8: Capital contribution = reserves + surplus – premium; capital distribution = reserves + surplus – losses; inner circle is required surplus + mandated reserves – fair value reserves.

Illustration: An emerging economy exempts shareholder dividends from personal income taxes and provides extensive disclosure of insurer accounts. If liquidity risks and principal agent problems are not material, the costs of holding capital may be low. The insurer prices its policies by a discounted cash flow model of underwriting cash flows, with only a small addition for the additional return demanded by investors.

Non-Cash Assets

Discounted cash flow models (NPV and IRR) consider cash flows only. They ignore non-cash transactions, such as deferred taxes and receivables.

Illustration: In 20X1, a firm expenses $100 million in post-retirement health care costs, but its taxable income does not recognize these expenses until they are paid. The firm has a deferred tax asset of $100 million × τ, where τ is the tax rate.
NPV considers the expenses when they are paid and the DTA when it is realized.
An insurance pricing model recognizes the DTA when it affects income. Non-cash assets affect pricing if they are of high enough quality to support reserves and surplus.

Illustration: An insurer begins operations on 1/1/20X1. On 12/31/20X1, it posts $800 million of full value loss reserves and $650 million of tax basis loss reserves. A tax rate of 35% gives it a 35% × ($800 million – $650 million) = $52.50 million deferred tax asset.

- U.S. GAAP income shows a DTA of $52.50 million.
- U.S. statutory income recognizes only a portion of the DTA.

If required capital is based on statutory RBC requirements, we use the statutory DTA. If required capital is based on rating agency capital adequacy standards:

- If the insurer had positive taxable income in recent years and expects positive taxable income in the coming years, a rating agency may consider the full DTA as a good asset and require less financial assets to meet its capital standards.
- If the insurer had negative taxable income in recent years and expects negative taxable income in the coming years, the DTA may never be converted into cash. The rating agency may require more financial assets to meet its capital standards.

Intuition: For industrial firms, capital flows from investors provide cash for fixed investments and net working capital. Deferred tax assets are not a substitute for cash. The capital requirements for insurers provide surplus to guard against statutory insolvency. Non-cash assets that can support reserves or surplus reduce the need for marketable securities, increasing the free cash flow to investors.

CAPITALIZATION RATE

The capitalization rate, or the opportunity cost of capital, is the expected return of investors in projects with similar risk. Investors diversify their investments, so only non-diversifiable risk affects the required return.

- Much investment risk is systematic and cannot be diversified. An insurer that invests in common stocks rather than bonds assumes a risk that its investors cannot diversify, so it should have a higher target return on capital.
- Most underwriting risk is not correlated with the risks of other firms and can be eliminated by diversification. Non-systematic risk does not warrant higher returns.

The effect of underwriting risk on the target return on capital is much debated.

- Financial economists say that natural catastrophes, workplace accidents, court awards, rate suppression, and other underwriting risks are peculiar to one industry and can be diversified by investors. In the 1970s, some financial economists measured systematic
underwriting risk as a residual risk after investment risk was accounted for. These underwriting betas were close to zero; differences were probably random fluctuation. Better understanding of the costs of holding capital have led economists to abandon efforts to find systematic underwriting risk.

- Some actuaries argue that insurance markets provide higher returns to riskier lines of business, whether or not this risk is correlated with overall market returns. They include risk loads for high-risk policies, and consumers pay the higher premiums. Some actuaries argue that insurance decreases the liquidity of capital funds. Investors are compensated for tying up their money in insurance enterprises.

Investment risk, underwriting risk, agency costs, double taxation, and illiquidity may affect the opportunity cost of capital, but we do not posit a specific relation. We assume the market return on equity capital is appropriate for insurers.

Actuaries have used two types of DCF models, relating the target return on capital on the investment yield.

- An investment yield reflecting expected returns, and a target return on capital expected by investors.
- An investment yield reflecting the risk-free rate, and a target return on capital reflecting illiquidity, agency costs, and systematic risk of underwriting. If these liquidity, agency, and risk costs are small, the target return on capital is close to the risk-free rate.

The second method is no longer used, since we can not measure illiquidity, agency costs, and systematic risk of underwriting. Most actuarial pricing uses the first method.
ILLUSTRATING THE PRICING PRINCIPLES

Insurance pricing adds two elements to discounted cash flows: required capital and the cost of holding capital. The excess of the policy premium over the present value of losses and expenses is the present value of the product of the required capital and the cost of holding this capital:

\[
\text{premium} - \text{PV(losses + expenses)} = \text{PV(required capital} \times \text{cost of holding capital)}
\]

The terms on the right side of the equation above can be expanded:

- Required capital = (surplus + required reserves – fair value reserves) \times \text{duration}^{46}
- Net cost of holding capital = required capital \times (opportunity cost of capital – after-tax investment yield)
- Cost of holding capital = net cost of holding capital \times \frac{1}{1 - \tau}, \text{where } \tau = \text{underwriting tax rate}

Required capital and the cost of holding capital differ by country. We explain the pricing principles for these differences, and then illustrate with the U.S. environment.

If no capital is required or the cost of holding capital is zero, insurance pricing is the same as discounted cash flow pricing for other firms.

TRENDS IN REQUIRED CAPITAL

Required capital depends on regulation, rating agency standards, and competitive markets.

- Solvency regulation (RBC requirements in the United States and Solvency II directives in Europe) set minimum capital requirements.
- Rating agency capital standards directly affect insurer behavior.
- The net requirements are measured by market premium to surplus ratios.

The capital embedded in reported reserves increased through 2010, but may decrease in the future.

- The excess of required reserves over fair value reserves increased over the past hundred years, as losses take more time to settle and regulators require full value reserves. This excess was justified as an implicit margin that raises reserve adequacy.
- The accounting profession dislikes implicit margins in reserves, which contravene the principle that assets and liabilities represent market values. Countries are adopting fair value for general accounting, with full value loss reserves replaced by the present value of unpaid losses. Solvency margins should be held as capital and surplus funds, not as liabilities. Even countries that permitted catastrophe and equalization reserves are
abandoning these liabilities and permitting only liabilities for the fair value of losses already incurred.

Stringent capital requirements offset the removal of statutory reserve margins. The U.S. moved from minimum surplus requirements to RBC standards in 1994. By the late 1990's, high rating agency target capital standards were a major influence on insurer capital management. Solvency II may require capital using principles-based economic capital models and a 99.5% value at risk.

Required capital levels are not easily measured. Insurers hold two or three times their RBC requirements to meet market pressures for financially strong firms. Insurers with excess capital may not distribute cash to stockholders for several reasons: to avoid federal income taxes, hold capital for rainy days, or expand the firm.

Regulatory and rating agency measures indicate required capital by line of business. We observe an insurer’s total capital, not its capital by line of business. The relative capital requirements from regulatory and rating agency standards are used to allocate the actual capital held by insurers by line of business. For example, if regulatory required capital is twice as high for medical malpractice as for personal auto (per dollar of premium), insurers’ actual capital may be allocated in the same proportion.

Insurance markets are competitive. In the long-run, the ratio of capital to premium is the ratio demanded by insurance markets. Insurers that hold too much capital earn lower returns; insurers with too little capital are downgraded and lose market share. Some economists believe the insurance industry is over-capitalized, so market leverage ratios may not be perfect indicators of required capital. But we have objective measures of over-vs under-capitalization, so market leverage ratios are the best estimates available.

The industry premium to surplus ratio has decreased over the past several decades, with temporary reversals reflecting catastrophes (Katrina in 2005; World Trade Center in 2001). Lower premium to surplus ratios reflect the higher capital demanded by rating agencies, which reflects the higher RBC requirements imposed by regulators.

**REQUIRED CAPITAL BY COUNTRY**

Adoption of fair value accounting by European insurers may reduce implicit capital held in reserves. In addition, the European Union speaks of *principle-based* RBC requirements, with insurers using internal capital models to determine surplus requirements.47

Higher capital requirements raise policy premiums and encourage insurers to seek lower cost jurisdictions. Some insurance tax havens (Bermuda, Cayman Islands) have low capital requirements as well. Capital requirements and tax laws affect pricing strategy. Insurers domiciled in these tax havens do not write much primary coverage in the U.S. or Europe, but they reinsure primary companies. If they reinsure affiliates, they convert high-cost regulated insurers to a less regulated ones.48
Market practice has followed theory. As high capital requirements increase the cost of regulation, insurers seek financial reinsurance with off-shore companies. Transferring unpaid losses to off-shore reinsurers that hold fair value reserves and little explicit capital and that provide letters of credit securing their reinsurance recoverables converts costs of regulation (required capital × the cost of holding capital) into the costs of security (letter of credit). If the cost of regulation is 7% of premium and the letters of credit cost 4% of premium, the reinsurance is profitable.

**COST OF HOLDING CAPITAL BY COUNTRY**

The cost of holding capital includes double taxation and financial friction costs (principal agent problems). These costs depend on the political and economic environments. Cash flows are readily estimated from historical experience, but the costs of holding capital are invisible and change from year to year. These costs depend on tax laws and the dynamics of corporate relations.

- If shareholders pay no tax on corporate dividends and if corporate and personal tax rates are the same on interest income, holding capital has no tax cost.
  - Investors would be indifferent between buying bonds directly or through an insurer.
  - High capital requirements for insurers would not affect policy premiums.
- If shareholders pay the same tax rate on corporate dividends as on other income, the tax cost of holding capital is high.
  - Investors are reimbursed by policyholders, so the costs of holding required capital is funded by insurance premiums.

The tax cost of holding capital has followed two trends over the past century. Populist politicians seek votes by raising taxes on wealthy shareholders; economists seek to curb double taxation and stimulate economic growth. The cost of holding capital in the U.S. increased from the early 20th century (the New Deal in post-Depression America) through the 1970’s, and then decreased with the Reagan-era tax cuts in 1980 and the decrease in the tax rate on stockholder dividends in the early 2000’s. European countries have diverse tax laws. With the fall of the Soviet Union, new market economies emerged in Eastern Europe, many of which sought tax regimes that did not discourage entrepreneurial activity. Some older European economies have followed suit, reducing double taxation.

The tax costs of holding capital are lower now than they have been in the past 25 years, both in the U.S. and in most European countries. The U.S. has a 15% tax on corporate dividends, and a lower rate for capital gains because of tax deferral. Some Western European countries with high tax rates do not tax shareholder dividends. Some eastern European countries have no double taxation at all.

Other financial friction costs are harder to quantify. In theory, managers seek to increase shareholder wealth; in practice, many managers pursue their own interests. Some managers prefer to enlarge their firms instead of distributing profits to shareholders. The
cost is reflected in the losses from failed acquisitions and other attempts by managers to appropriate shareholder funds. The friction costs vary by country:

- Lower in the U.S., with its strong ideology on increasing shareholder wealth.
- Higher in emerging economies with limited shareholder rights.

Investors suffer less from double taxation in emerging economies, but they are reluctant to entrust funds to corporate managers without the accounting and regulatory safeguards common in western countries.

Financial friction costs are estimated by the market risk premium by country. Suppose the market risk premium for common stock is 7% in the U.S. and 9% in an emerging market, and the personal income tax differential between capital gains and other income is the same in the two countries. We presume the higher friction costs in the emerging market account for the higher market risk premium.
TAXES VS COST OF HOLDING CAPITAL

Pricing with after-tax cash flows is separate from double taxation. The distinction between implied capital flows and the insurer’s cash flows applies even to after-tax cash flows.

Principle #1: Pricing uses after-tax cash flows. Even if investors contribute no capital or their cost of holding capital is zero, we price policies with after-tax cash flows.

Principle #2: If taxable income reflects economic income and investors contribute little capital, pre-tax income is small in a competitive market with no systematic risk.

Principle #3: A high tax rate does not raise the policy premium if shareholders provide no capital or if their cost of holding capital is zero.

Principle #4: Positive required capital and a cost of holding capital increase the policy premium in two stages:

- Policyholders reimburse shareholders for the cost of their capital.
- This reimbursement is taxed as underwriting profits.
Insurance Pricing Examples

Exercise 1.1: No Capital flows: Fair Value with no Surplus

If insurers hold no capital, the premium is the present value of losses and expenses. Taxes are relevant if taxable income differs from economic income. Even though unpaid losses are risky, the fair value risk margin is zero, since the risk is diversifiable. Suppose

- Insurance reserves are the present value of future obligations.
- Insurers hold no surplus; they pay losses and expenses from policy premiums.
- To avoid investment risk, insurers invest in risk-free securities.
- Taxable income is economic income.
- Insurance markets are competitive.

In competitive markets, firms earn zero economic income. If insurers hold no capital, the cost of holding capital does not affect pricing.

- The pre-tax income is zero, so the tax liabilities are zero.
- The tax rate does not affect premium if taxable income reflects economic income.
- The policy premium is the net present value of future losses and expenses.

Profit is earned on invested capital. If the insurer invests no capital, expected profit is zero.\(^49\)
Exercise 1.2: Zero Cost of Holding Capital

If the cost of holding capital is zero, the fair value risk margin is zero. Suppose

- A country levies a 30% tax on corporate profits and labor income and exempts asset income (including shareholder dividends) from personal income taxes.
- Corporate managers serve shareholder interests, and financial friction costs (principal agent problems) are not material.
- Taxable income is economic income.
- Insurers hold gross unearned premium reserves and full value loss reserves.
- Required surplus is 50% of written premium.

How does the tax rate affect the policy premium? If the tax rate changes from 30% to 40%, does the premium for insurance policies increase, decrease, or stay the same?

Solution 1.2: The cost of holding capital (double taxation and friction costs) is zero.

- Investors pay the same tax whether they invest money directly or through the insurer.
- Policyholders pay the cost of insurance operations, not the tax on investment income of investors' funds.

⇒ The change in the tax rate does not affect the policy premium.
**CAPITAL REQUIREMENTS AND COST OF HOLDING CAPITAL**

In practice, insurers face capital requirements and costs of holding capital. Even a country with fair value accounting, no RBC requirements, and no taxes has rating agencies that impose capital standards and financial friction costs of holding capital. Accounting rules derive the fair value risk margins. But if the risk margins do not affect tax liabilities or required capital, the FASB accounting rules affect the presentation of capital as liability or equity, not the total capital.

Return on capital pricing reflects capital requirements, tax law, and other financial friction costs. We use the U.S. environment for an illustration, since its reserve requirements, RBC charges, and tax laws are well known and have not changed in fifteen years. These are:

- Insurers hold gross unearned premium reserves and full value loss reserves.
- RBC requirements are specified percentages of written premium and held reserves.
- Investors incur double taxation on shareholder dividends.

The illustration focuses on pricing principles, not details: how capital requirements and costs of holding capital affect the policy premium and the fair value risk margin.
Capital Flow Graphic

Insurance pricing depends on reserves and surplus requirements that vary by country. We focus on the pricing principles: separating the present value of losses and expenses from the present value of the profit load.

The graphic below shows policyholders on the right and investors on the left.

- Policyholders pay the fair value of losses to claimants and related expenses.
- Investors’ funds support insurance operations and earn investment income.

Were there no costs of holding capital (leakage to tax authorities), the policy premium for the policyholder / insurer / claimant transactions on the right side would not be affected by investor transactions. But the leakage to tax authorities #1 (on the top of the Figure) must be paid by policyholders. This payment is a second leakage (on the bottom of the Figure).

![Figure 9](image)

**Figure 9** If the cost of holding capital = 0, policyholders fund the insurance operations on the right and investors fund the capital flows on the left. If cost of holding capital is more than zero (leakage to tax authorities #1), policyholders pay this cost (bottom arrows), incurring additional leakage to tax authorities #2.

The figure above shows premium, expenses, losses, taxes, required capital, and the cost of holding capital. The cash flows from a policy are

- CU 10,000 of gross premium (not including reimbursement for cost of holding capital)
• CU 2,500 of acquisition expenses
• CU 9,000 of losses
• CU 1,500 of investment income

The cost of holding capital is 4% per annum and the corporate tax rate is 33.3%.

Step #1: The insurer’s cash flows with policyholders and claimants are in the top semi-circle of boxes, from policyholders on the left to claimants on the right. If no capital is needed for statutory reserves or surplus requirements, the policy is priced by ordinary NPV techniques, using either of the equations below:

- Premium + investment income = (nominal) losses + expenses
- Premium = PV(losses + expenses)

If taxable income equals economic income, pre-tax income is zero and the tax liability is zero. Income taxes are paid on profits, and are a percentage of invested capital. With no required capital, profit is zero and taxes are zero.

Contributions from and dividends to shareholders are zero. Losses and expenses are paid from the policy premium. The cash flows from insurance transactions do not affect shareholders, and shareholder tax rates do not affect policyholders.

Step #2: Add capital requirements: gross unearned premium reserve, full value loss reserves, and surplus equal to half of written premium and a third of loss reserves. These are the dashed boxes in the figure.) The money comes from shareholders and is returned to shareholders. The required capital depends on insurance transactions, but if the cost of holding capital is zero, the policy premium is unaffected.

**Principle:** If the costs of holding capital are zero, the capital flows are separate from the cash flows. Shareholders contribute capital at policy inception, earn investment income on the capital, and receive their capital back (with investment income) as the policy expires and losses are paid.

Step #3: Add a 4% *per annum* cost of holding capital: double taxation and other financial friction costs (such as principal agent problems). During the policy term, the insurer holds CU 7,500 of investor capital, at a cost of 4% × CU 7,500 = CU 300.50 During the next year, the insurer holds CU 4,000 of investor capital, at a cost of 4% × CU 4,000 = CU 160.51 The figure shows the cost of holding capital as *leakage* to tax authorities or financial friction.

Policyholders pay the cost of holding capital, grossed up for another layer of leakage.

- If policyholders directly reimbursed shareholders, they would pay CU 300 the first year and CU 160 the second year.
- In fact, policyholders reimburse the insurer for the leakage, which forwards the money to its shareholders. The tax authorities view this reimbursement as profit, so the insurer...
incurs a tax liability. If the corporate tax rate is 33.3%, the cost to policyholders is $1\frac{1}{2}$ times the original leakage: $1\frac{1}{2} \times (1 - \frac{1}{3}) = 100\%$. 


**Pricing Principles and Illustrations**

The policy premium comprises (i) the fair value of losses + expenses and (ii) a profit load. The fair value of losses and expenses is their nominal value discounted at a risk-free rate. The profit load is derived by the following steps, computed for each valuation period.

- Required capital is required surplus, as shown by rating agency standards or industry premium to surplus ratios + (statutory reserves – fair value reserves).
The cost of holding capital is determined from tax law and market risk premiums for holding common stock.
The leakage is the required capital × the cost of holding capital.
The policyholder reimbursement is the leakage / (1 − τ), where τ is the tax rate.
The profit load is the present value of the reimbursement for each valuation period.

The illustrations show the effects of reserve valuation, capital requirements, and taxes on the indicated policy premium. The illustrations focus on the concepts explained in this study note, not on the details of modeling.

Endnotes:

1. The suppliers of capital may be termed the owners, the investors, the shareholders (or stockholders), or the equityholders; we generally use the term "equityholders" in this paper. Mutual insurance companies do not have stockholders or investors and their policyholders have only limited ownership rights, but they face the same opportunity cost of capital as stock companies face.

Richard Goldfarb points out to me that a pure cash flow perspective might seem incongruous with global financial crisis of late 2008, when giant banks and investment firms were devastated by paper losses from derivative securities. Economists do not agree if the reductions in asset values reflect

1. Temporary liquidity problems or permanent credit defaults.
2. Expected changes in the long-term cash flows from these securities.

2. The pricing characteristics of insurers are true for other regulated financial institutions as well, such as banks, savings and loans, and investment firms.

3. In a review of this study note for the CAS Syllabus Committee, Quan Shen notes that implied capital flows (the free cash flows to the firm) gives the same pricing indication as implied equity flows (the free cash flows to investors) if capital markets are perfectly efficient. With market imperfections (corporate income taxes, costs of bankruptcy, and agency costs), it is useful to examine both approaches. The WACC changes with financial leverage, giving a premium rate that depends on costs of bankruptcy and agency costs.

4. To distinguish between the company cash flows and the flow of funds to suppliers of capital, we refer to the former as "cash flows" and to the latter as "implied capital flows." The financial community uses the term "free cash flow" instead of implied capital flow. Atkinson and Dallas [2000], chapter 11, use the term "distributable earnings" instead of implied capital flows. The pricing model here is similar to the Atkinson and Dallas life insurance pricing model, with differences in reserve requirements and federal income.
tax liabilities.

The Actuarial Standards Board, “Actuarial Standard of Practice No. 19: Actuarial Appraisals” (October 1991), page 4, has the same view of distributable earnings: “5.2.1 Distributable Earnings – For insurance companies, statutory earnings form the basis for determining distributable earnings, since the availability of dividends to owners is constrained by the amount of accumulated earnings and minimum capital and surplus requirements, both of which must be determined on a statutory accounting basis. Distributable earnings consist of statutory earnings, adjusted as appropriate to allow for the retention of a portion thereof or the release of a portion of prior accumulated earnings therein, in recognition of minimum capital and surplus levels necessary to support existing business. . . . Economic value generally is determined as the present value of future cash flows. Statutory accounting determines the earnings available to the owner. Hence, while future earnings calculated according to generally accepted accounting principles (GAAP) will often be of interest to the user of an actuarial appraisal, as may other patterns of earnings, the discounted present-value calculations contemplated within the definition of actuarial appraisal in this standard should be developed in consideration of statutory earnings, rather than some other basis. . . . The actuary’s report should include a discussion of factors, such as capital needs (whether perceived by the actuary or imposed by an external entity such as a regulator), that may cause the earnings available for shareholder or policyholder distribution to be different from statutory earnings.”

5. Other industries may have large fixed costs and fluctuating demand; future cash flows depend on business cycles and market competition and are difficult to forecast.

6. Fn: The financial pricing model described here applies a target return on capital (based on the insurer’s opportunity cost of capital) to book values of capital. But the cost of capital applies to market values. An insurer’s market value is its book value plus its franchise value – the intangible worth of the insurer’s brand loyalty, distribution system, underwriting experience, and market penetration; see Goldfarb [2007].

- New insurers have lower returns, as they charge low rates to break into new markets and attract lower quality consumers who are not desired by incumbent insurers. This low book return may be deceptive, since the insurer is building its franchise value.
- Established insurers have higher returns, as they earn profits from renewal books of business with low loss costs and expense ratios. This high book return may be deceptive, since the insurer starts from a high market value and is using up the franchise value built up in previous years.

Illustration: Suppose insurers have 12% opportunity costs of capital.

- In perfectly competitive markets, each insurer earns a 12% return on capital.
- This return includes
  ○ the insurer’s franchise value as part of capital (in the denominator).
the change in franchise value as part of income (in the numerator).

For policy pricing, we use book values of capital, not including franchise value. The return on individual policies may be more or less than 12%.

- Writing new business is expensive, since loss costs and expense costs are higher for new business than for renewal business.
  - But writing new business builds up the franchise value of the insurer.
  - The expected IRR for new business is less than 12%, since the IRR does not consider the change in franchise value.
- Writing renewal business is profitable, since loss and expense costs are low.
  - Writing renewal business uses up some franchise value.
  - The expected IRR is more than 12%, since the IRR does not consider the reduction in franchise value in the numerator
  - or the existing franchise value in the denominator.

7. Modern portfolio theory says that investors are not rewarded for assuming diversifiable risk in efficient (arbitrage free) markets. The rationale is that if assets with high unique (i.e., diversifiable) risks received high returns, a mutual fund comprised of these assets would receive the high return even though the unique risks were eliminated by diversification.

The CAPM (Capital Asset Pricing Model) says that the excess of the investment yield over the risk-free rate is proportional to covariance of the asset returns (or project returns) with the overall market return. Some pricing actuaries (particularly for high-risk reinsurance treaties) add risk loads to cover the high variance in the total return, not solely the covariance with market returns. See Kreps [2000] for a pricing method that combines IRR modeling with actuarial risk load concepts and Goldfarb [2007] for a discussion of insurance RAROC (risk adjusted return on capital).

Before the 1990s, the CAPM was widely accepted as the measure of systematic risk and its return. But the CAPM has fared poorly in rigorous empirical tests, such as those by Fama and French [1992]. This study note does not assume the CAPM or any other asset valuation model is correct.

8. In practice, transactions with capital providers occur infrequently, since all the firm's projects are aggregated and it is inefficient for capital providers to continually contribute or receive cash.

9. Some actuaries object that the capital flows are not real, based on the following reasoning:

   We speak of an implied capital flow to fund the underwriting loss at policy inception. But there is no actual capital contribution when a policy is written. In contrast, the
company cash flows used in other industries are actual transfers of cash.

This objection is specious. The implied capital flows are real, though they are submerged under a multitude of policies and the other capital structure decisions of the company.

Suppose a reinsurer writes $100 million of casualty excess-of-loss reinsurance on January 1, 20X1, with capital requirements of 25% of premium. The pricing model uses an implied equity out-flow of $25 million on January 1, though there is no actual flow of funds from capital providers. The pricing perspective is that

- $25 million of capital supporting the premium risk in 20X0 is transferred to support the new year's writings.
- The pricing model shows a $25 million return of surplus for the 20X0 business and a $25 million investment for the 20X1 business.

If the reinsurer’s capital requirements increase, it may write less business or curtail other activities. If its capital requirements decline, it may pay stockholder dividends, buy back stock, or use the capital for other activities.

The premium to surplus leverage ratios for the insurance industry have been declining steadily, though catastrophes like Hurricane Andrew and the World Trade Center incident cause short, offsetting spikes. One cause is the heightened emphasis on capital adequacy by state regulators and rating agencies. (Other causes include the maturing of the industry in an era of declining demand with many suppliers.) Some regulators presume that stringent capital requirements help consumers, by preventing insolvencies and forcing weak insurers from the market. But higher capital leads to higher premiums, with the additional money going to the IRS. Neither the insurance industry nor its consumers gain from excess capital.

The effects of implied capital flows are not immediate, but eventually they are realized. The pricing model attributes these capital flows to the policies that require the capital.

10. This study note uses RBC concepts since actuarial candidates know the subject from the CAS syllabus. Publicly traded insurers may be more concerned with Standard and Poor’s or Moody’s capital adequacy standards, since a ratings upgrade or downgrade may affect the stock price. Mutual insurers and non-publicly traded stock insurers may place more weight on the A.M Best’s capital formula.

11. The table below shows the signs of the cash flows and implied capital flows for insurance operations. A positive sign is an inflow and a negative sign is an outflow; N means no cash flow or capital flow.
Illustration: A policy premium of $1,000 is written and collected on December 31. The agent’s commission is 20%, and the capital requirements are 25% of premium.

- The net cash inflow from the policyholder is $1,000 – $200 = $800.
- The tax liability is zero and the deferred tax asset is $70 (see below).
- The company receives cash of $800, and it holds $1,000 of unearned premiums and $250 of surplus. The equity outflow is $1,000 – $70 – $800 + $250 = $380.

12. Rating agencies have no intrinsic influence. They derive their influence from capital and product markets. A publicly traded commercial lines insurer that seeks to issue debt may pay great heed to Standard and Poor’s capital adequacy standards, whereas a mutual personal auto writer with its own agents and no debt may be more concerned with RBC requirements.

13. Fair value is the present value at a risk-free rate plus a market based risk margin if appropriate. It reflects the market value the loss reserve would have if it were traded in a liquid market between two willing and independent investors. No consensus yet exists on the fair value risk margin for insurance liabilities, if any.

- Many financial economists believe no risk margin is appropriate, since underwriting risks are diversifiable.
- Many actuaries and insurers believe the fair value risk margins are material, based on the cost of holding capital for loss reserves or the uncertainty in loss payments.

The author of this study note has published his views on fair value risk margins in other papers. For this study note, fair value means present value at a risk-free rate. The risk margin is part of the cost of holding capital.

14. NAIC IRIS tests use premium to surplus ratios. Reserves to surplus ratios are used in Great Britain and some other countries, as pointed out to me by Ralph Blanchard.

15. Policyholder reserves are unearned premium, loss, and policyholder dividend reserves.
16. Net working capital not included in reserves or surplus is the non-admitted value of buildings, office furniture and equipment, training costs for staff, non-admitted receivables, and the provision for reinsurance. The insurer spends money for these items, but they do not support policyholder liabilities.

17. Some early property-casualty pricing models overstated the returns by erroneous timing of the implied capital flows. Similarly, early cash flow pricing models assumed the cash flow is book income adjusted for non-cash revenues and expenditures. A firm with sales of $10 million, of which $2 million are receivables, has $8 million of cash inflows. This presumption is no longer used, because many cash flows – such as the purchase of supplies – are not revenues or expenditures.

To illustrate this, consider a non-leveraged (all-equity financed) firm that manufactures aircraft. The firm leases the factory, equipment, and workers, so capital expenditures are zero. To simplify the computations, we assume that rent and wages are paid at year end.

- At time 0, the firm purchases material and supplies for $10 million.
- From time 0 to 4, manufacturing costs are $2 million a year in rent and wages.
- At time 5, the firm sells the aircraft for $25 million and pays $1 million in commissions.

There are no interest payments, debt payments, or depreciation; earnings are

- $–2 million at times 1, 2, 3, and 4 (production expenses).
- +$14 million at time 5: sales ($25 M) – cost of goods sold ($10M) – expenses ($1M).

At time 0, $10 million of cash is exchanged for inventory. The inventory changes in form over the next five years, but there is no revenue until time 5. If the cost of equity capital is 15% per annum, the apparent NPV of this project is (in millions of dollars)

\[-$2 / (1.15) – $2 / (1.15)^2 – $2 / (1.15)^3 – $2 / (1.15)^4 + $14 / (1.15)^5 = $1.25.\]

The equation above does not consider the change in net working capital (inventory). The cash outflow from capital providers at time 0 is the $10 million net working capital increase. At time 5, the $10 million decrease in net working capital is an additional cash inflow to the capital providers. The corrected net present value of this project is (in millions of dollars)

\[-$10 – $2 / (1.15) – $2 / (1.15)^2 – $2 / (1.15)^3 – $2 / (1.15)^4 + $24 / (1.15)^5 = –$3.78.\]

The capital providers’ perspective turns a $1.25 million profit into a $3.78 million loss.

18. For simplicity, we use annual valuations here. In truth, the unearned premium reserve is taken down continuously during the year, and expected losses occur
continuously during the year.

19. We have ignored investment income, changes in required capital, current tax liabilities, and deferred taxes, to focus on the difference between cash flows and capital flows.

20. Actual fair value accounting does have risk margins and capital requirements. No required capital means the insurer has no reason to hold capital, whether regulators or rating agencies demand capital or shareholders want to hold capital. Insurers hold capital for many reasons, not just regulatory or rating agency requirements. This was clearest before the advent of U.S. risk-based capital requirements in 1992-1994. State minimum capital requirements were a few million dollars, but large insurers held billions of dollars in capital. See Panning [2005] for why investors may want insurers to retain excess cash.

21. For simplicity, we use annual valuations here. In truth, the unearned premium reserve is taken down continuously during the year, and expected losses occur continuously during the year.

22. Ralph Blanchard stresses that the IAA and the American Academy of Actuaries use this view.

23. Ira Robbin’s 1992 CAS study note on the underwriting profit provision takes this perspective.

24. Available assets may include non-cash items (deferred tax assets, accrued interest, receivables) if they are of acceptable quality to back liabilities and required capital. Insurance liabilities include deferred tax liabilities and accounts payable. Ralph Blanchard points out a major advantage of this method: if accounting rules and required surplus change by offsetting amounts, total assets do not change.

25. The Annual Statement shows statutory income and the change in statutory surplus. The figures are readily available and are understood by insurance personnel.

26. Free cash can be taken by capital providers; cash inventories are locked up in the firm. The relations between insurance and generic pricing models are summarized below.

<table>
<thead>
<tr>
<th>Component</th>
<th>Generic Pricing Model</th>
<th>Insurance Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>term for implied capital</td>
<td>free cash flows</td>
<td>distributable earnings</td>
</tr>
<tr>
<td>proxy for implied capital</td>
<td>cash flows</td>
<td>statutory earnings</td>
</tr>
<tr>
<td>short term investment</td>
<td>inventory (net working capital)</td>
<td>reserve requirements</td>
</tr>
<tr>
<td>taxes</td>
<td>current tax liability</td>
<td>accrued taxes: liability ± capital</td>
</tr>
<tr>
<td>long-term investment</td>
<td>capital investment</td>
<td>capital requirements</td>
</tr>
</tbody>
</table>
27. The comparison in the text has exceptions. An direct writer may build up a distribution system over several years, using cash to subsidize agents’ offices, similar to an auto manufacturer subsidizing a dealership. But financial service firms (insurers, banks, brokerage houses) have lower sunk costs and shorter production cycles than most industrial firms. Reserve and surplus requirements, however, take the place of inventories and fixed investments.

28. Some analysts have mistakenly tried to model insurance operations as loans from policyholders. But a borrower gets cash that can be used for other purposes; the insurer must hold all the cash plus additional capital to support policyholder reserves.

29. Illustration: A supermarket needs $500,000 of cash to stock its shelves and storerooms, and $2,500 for its cash registers. For pricing models, the cash inventory is subsumed under inventories of goods: $502,500 is needed to stock the store.

30. We use the term buy cash metaphorically. The insurer simply designates the cash as backing the UEPR.

31. FN: Insurers do not show long-term debt on their balance sheets, and the CAS exam syllabus does not discuss insurers’ capital structure. Readers may wonder if the weighted average cost of capital differs from the cost of equity capital. We examine

- The traditional reasons for insurers’ all-equity financing.
- The exceptions for non-stock (mutual) insurers.
- The rationale for debt financing by other financial institutions.
- The methods of debt financing now used by insurers.

Debt provides cash for manufacturers to build plants, buy equipment, and produce goods. Manufacturers raise cash from bank loans and corporate bonds for investments made years before their products are sold.

Insurers collects premiums before paying losses, and they need no equipment to produce insurance policies.

- Their surplus guarantees their promises to policyholders and claimants.
- Debt – with a prior lien to bondholders – is no guarantee of insurance obligations.

Mutual insurers do not have shareholders. They are funded initially by surplus notes issued with the approval of the domiciliary insurance commissioner. Coupon payments on surplus notes are made from earnings. They are contingent on the approval of the commissioner and are subordinate to the claims of policyholders and claimants. Capital notes are similar to surplus notes and can be issued by all insurers.
Banks make similar long-term promises to depositors, and they have similar RBC monitoring as insurers. RBC for commercial banks allows long-term subordinated debt as Tier II capital. Debt with at least five years remaining to maturity and subordinate to depositors’ claims is included at 50% value for RBC capital.

Insurers use holding companies, affiliates, or subsidiaries to issue debt.

Illustration: An insurer has $2 billion of surplus and needs $2.5 billion to meet rating agency requirements for its desired rating. Issuing stocks to raise equity capital generally leads to a stock price decline that eliminates (on average) 30% of the new capital raised. Instead, the insurer forms a non-insurance holding company that owns the insurer and is owned by the original investors. The holding company issues $500 million of debt, and it gives the cash to the insurer as equity capital.

All these forms of insurer debt are subordinate to policyholders and claimants. In the holding company illustration above, if the insurer does not have $500 million of excess cash when the debt comes due, it can not pay a $500 million dividend to its parent (the holding company).

32. One is tempted to say that cash flows are important because of the time value of money. Cash earns the opportunity cost of capital, whether this is interest income or an equity return. One might reason that equity in the bank earns interest, whereas a pile of dollar bills serving as an inventory of cash, like currency in a cash register, does not earn interest.

This is not correct. The unearned premium reserve is an accounting entry. The cash is not physically removed from the bank account and placed in an inventory. The insurer makes an accounting entry showing that cash available to owners is transferred to an inventory of cash. The cash remains in the bank, and the interest received does not change.

One is tempted to say that the cash available to owners can be placed in any investment: a mutual fund, stock portfolio, real estate, venture capital, or high yield bonds. The cash in the unearned premium reserve and loss reserve is in lower yielding securities: Treasury bonds, corporate bonds, and municipal bonds. But this is not correct; an insurer can invest its cash in any of these investments. The NAIC model investment act restricts equity investments and derivatives to a fraction of the insurer’s assets or surplus. But most insurers hold less equities and derivatives; the model act is not constraining. Similarly, risk-based capital requirements make equities more expensive for insurers, since they must hold additional equityholder capital to support them. But this capital charge is low for property-casualty insurers after the covariance adjustment in the RBC formula.

One is tempted to say that buying the inventory of cash exposes the insurer’s owners to the risk that they may not receive their cash back if the insurer becomes insolvent. But
An insurer faces two types of risk: investment risk and underwriting risk.

- The investment risk does not change when the accountant transfers the cash notionally from a stock fund representing free cash to a stock fund backing policyholder reserves. The fund’s return compensates for the additional risk, regardless of who holds the fund.
- Underwriting risk is diversifiable, with no effect on diversified capital providers. Systematic risk from insurance operations is rewarded by a higher profit margin in the premium.

Ralph Blanchard points out that the American Academy of Actuaries says that changes in accounting rules affecting the capital embedded in premium and loss reserves should be offset by changes in surplus requirements that leave total assets unchanged.

Double taxation affects all firms, but the effects are greatest on property-casualty insurers. The IRS taxes nominal interest income, with no adjustment for inflation, but it does not tax the unrealized appreciation of fixed assets or LIFO inventory. Many firms with large fixed assets use debt financing, which avoids the double taxation costs entirely. For example:

- A manufacturer uses $2 billion of capital, half equity and half debt, to build a plant, which it depreciates over 20 years. The firm writes off $100 million in tax depreciation and $100 million in debt payments each year, though the market value of the plant may be rising 5% a year because of inflation and has a useful life longer than 20 years. Investing cash in fixed assets avoids or defers the tax costs of inflation.
- An insurer invests its capital in bonds yielding 8% per annum. If inflation is 6%, the true return is 2% per annum. The insurer pays 2.8% of the yield in corporate income taxes, which is grossed up to a profit margin of 2.8% / (1 – 35%) = 4.31% in the premium. The tax is twice as great as the real interest income. The owners then pay personal income taxes on the dividends they receive from the insurer.

We divide the corporate tax on equityholder provided capital into four investment income pieces and an additional layer of underwriting income tax. The investment income on equityholder provided capital stems from:

- The equity in the unearned premium reserve
- The implicit interest discount in the loss reserves
- The capital in surplus supporting the written premium
- The capital in surplus supporting the reserves
These are grossed up by $1/(1 - \tau)$ for pricing, where \tau is the underwriting tax rate.

Some readers say that the actuary’s task is to quantify insurance risks and determine the appropriate return for them; taxes are an annoying but minor item that should be handled by a tax accountant. The layers of insurance taxation form the dominant portion of the profit margin in the premium; underwriting risk is diversifiable and warrants little reward.

Some actuaries presume that the tax cost is theoretical, imagined by financial economists but not truly present. “The tax is 35% of income,” they say, “regardless of the protestations of financial economists.” But this is not correct. The tax liability is 60% to 70% of the profit margin in the premium, regardless of the protestations of pricing actuaries.

Some pricing actuaries protest that we should not include the personal income taxes paid by investors. That is correct; the 60% to 70% is just the corporate income taxes. If we include personal income taxes paid by investors, the total tax rises to 75% of the profit margin in the premium. We do not include the personal income taxes to price the policy.

36. If the receivables are valuable, they may serve as security for creditors. But the creditors focus on the anticipated receivables; the recognition of the deferred taxes in financial statements is not relevant.

37. Even for other firms, a merger or acquisition may reflect deferred tax assets that could not be used by the acquired firm. Yet deferred taxes are not used for pricing:

- Deferred taxes affect future cash flows. If the tax cash flows are modeled separately from book income, deferred taxes are not relevant.
- For many firms, deferred taxes are small and offsetting. The firm may have deferred tax liabilities from stock appreciation or accelerated tax depreciation and deferred tax assets from more rapid recognition of post-retirement health benefits in GAAP statements. The net deferred taxes may not be material.
- Most deferred taxes are not related to specific operations and can not be anticipated in product pricing.
- Deferred taxes are specific to tax accounting. Deferred tax liabilities from accelerated depreciation depends on tax depreciation schedules. Financial analysts ignore the effects (as not material) or ask the tax officers at their firms to provide figures; they do not develop deferred tax modules in pricing models.
- For a non-insurance firm, all deferred taxes are recognized if they are expected to be paid or collected. Accounting admissibility of deferred taxes is not an issue.

Each item above is reversed for property-casualty insurers.

- Deferred taxes contribute to distributable earnings.
Deferred taxes are large and not offsetting. Deferred tax liabilities are small (unless the insurer has a large common stock portfolio), whereas deferred tax assets are large, about 4% to 8% of statutory surplus for many insurers with long-tailed lines of business.

- The major deferred tax assets stem from underwriting operations: premium writing and loss incurral. The deferred tax asset from revenue offset is about 7% of unearned premium reserves or about 2% to 3% of statutory surplus. It is accrued whenever an insurer writes premium, regardless of the loss or expense costs. The gross deferred tax asset from loss reserve discounting is about 5% of the full value loss reserves for long-tailed lines of business or about 10 to 15% of statutory surplus. It is accrued whenever the insurer incurs losses. It depends on IRS loss reserve discount factors and is easily modeled.

- The deferred tax assets stem from two provisions of the 1986 tax law. They are the same for all insurers. They are calculated from the tax flows and loss payment patterns already used in the pricing model.

- The admissible part of the deferred tax asset from loss reserve discounting is an actuarial calculation. The actuary provides this figure, not the tax accountant.

For other firms, deferred taxes accelerate or postpone the tax liability. If taxes of $1,000 are postponed a year, creating a $1,000 deferred tax liability, the savings is the investment income on $1,000. This savings is already considered by the cash flow model, which uses the payment dates of the tax.

For an insurer, a deferred tax liability offsets the reduction in capital requirements caused by lower taxes, and a deferred tax asset offsets the increase in capital requirements caused by higher taxes. Deferred tax assets for insurers are large, and proper modeling of deferred taxes is important.

38. Fairley found a $\beta$ for insurance liabilities of $-0.21$, similar to the CAPM beta for common stocks, but his results could not be replicated by other researchers.

39. Readings by Miccolis, Bault, Kreps, and Mango on the CAS Exam 9 syllabus reflect the actuarial view that the equilibrium underwriting return is proportional to the standard deviation or the variance of surplus. Ralph Blanchard, on reviewing this IRR reading for the CAS syllabus, emphasized that the financial perspective linking return to systematic risk only is not borne out by insurance markets.

40. Empirical correlations between underwriting risk and return are hard to replicate: correlations found by one analyst are absent in other studies.

41. Suppose an insurer invests in Treasury bills only, pays no taxes, and has no agency costs or systematic underwriting risk.

- The market for Treasury bills is liquid. The bills can be sold readily at their fair value.
- A investor to an insurer that buys Treasury bonds accepts a liquidity risk as well.
The insurer may pay low dividends, or it may repay its corporate debt over thirty years. Underwriting cycles, catastrophes, and financial crises may further disrupt markets in insurance stocks and bonds.

The investors exchange liquid Treasury bonds for less liquid securities.

- The investor may require another 100 – 200 basis points for the lower liquidity.
- This liquidity premium is part of the cost of holding capital.

42. Liquidity premiums are hard to measure. Some economists quantify the liquidity premium as the yield spread for non-investment grade bonds minus the additional default premium.

*Illustration:* A Bb bond has a 450 basis point spread over Treasury bonds. If its higher defaults warrant a 300 basis point spread, it is presumed to have a 150 basis point liquidity spread.

43. Some financial economists view insurers as leveraged investment trusts to estimate the target return on capital. Suppose the insurer holds 20% common stock replicating the overall market and 80% investment grade corporate, municipal, and Treasury bonds. The risk-free rate is 7% per annum, the market risk premium is 5%, and the insurer’s assets to capital ratio is four to one. The systematic risk of the asset portfolio is $20\% \times 5\% = 1\%$, so the systematic investment risk affecting equity is $4 \times 1\% = 4\%$. Liquidity risk, underwriting risk, and agency costs may add another 2% to the target return on capital, giving $7\% + 4\% + 2\% = 13\%$.

44. Short term Treasury bills were once used as the risk-free rate, but they have state tax exemptions and are much demanded by foreign states, so their yields are low. Financial economists now use duration matched LIBOR as risk-free rates, where the loan term has the same maturity as the average loss reserve.

45. The second method was pioneered by William Fairley, Stuart Myers, and Richard Cohn, who developed pricing models for state-made insurance rates in Massachusetts. Their models were appropriate (perhaps) where the state set uniform premium rates but did not mandate investment practice. Their pricing models estimated the risk from underwriting and the appropriate return on underwriting alone, not on investments.

But their models did not quantify the systematic risk of underwriting and the appropriate return. Instead, the models backed out the investment risk from the systematic risk of insurance and assumed the rest was the systematic risk of underwriting. Their data were sparse and could not be analyzed by line of business, so their models were inaccurate and prone to distortions by line. For an internal financial pricing model used by a single insurer, this method has no advantages.
Attempts to relate the presumed systematic risk of underwriting to premium or reserves led to further distortions. A premium base overstated the risk of property lines and understated the risk of casualty lines; a reserves based did the opposite.

Using a risk-free rate as the investment yield and the market return on insurers as the target return on capital incorrectly mixes the two methods and overstates the required underwriting profit. The systematic investment risk is twice compensated: the actual investment yield exceeds the risk-free rate and the underwriting profit margin is raised to yield the higher target return on capital.

46. The duration is the time the capital is held, not the technical meaning used in bond valuation.

47. Rating agencies tend to use more lenient capital standards in countries with more lenient regulation. If regulators accept discounted reserves or 3:1 premium to surplus ratios, rating agencies do not object. As rating agencies develop economic capital models, they may use similar standards across countries.

48. In some countries, tax or regulatory officials may not allow the benefits. For example, the IRS does not permit firms to reduce their tax liabilities by actions that have no business purpose.

49. In the U.S., taxable income is close to economic income for the average policy. In certain scenarios, taxes may be paid faster or slower. On high deductible policies, for instance, the IRS loss reserve discount is less than the true discount, so taxes are paid faster, and the present value of after-tax cash flows increases. Shareholders fund the pre-payment of taxes, increasing required capital.

If an insurer invests in common stocks, the higher expected return is treated as profit and taxed. The premium is the present value of future losses, expenses, and taxes.

50. If we use continuous functions instead of annual valuations, the average expense cost is \( \frac{1}{2} \times (CU\ 2,500 + CU\ 0) = CU\ 1,250 \). The average capital held during the year is CU\ 6,250, and the cost of holding this capital is 4\% \times CU\ 6,250 = CU\ 250.

51. If we use continuous functions instead of annual valuations, the average implicit interest discount is \( \frac{1}{2} \times (CU\ 1,000 + CU\ 0) = CU\ 500 \). The average capital held during the year is CU\ 3,500, and the cost of holding this capital is 4\% \times CU\ 3,500 = CU\ 140. If the loss is incurred midway through the policy term (not at year end), the cost of holding this capital is 1.5 \times 4\% \times CU\ 3,750 = CU\ 225.
**ATTACHMENT B: PRICING ILLUSTRATION FOR IASB DELIBERATIONS ON FAIR VALUE**

**ILLUSTRATION A: PREMIUMS**

The illustration below uses U.S. capital requirements and tax law. Tax basis loss reserves are approximately the present value of unpaid losses at a risk-free rate for most blocks of business, though not for sub-lines with payment patterns that differ from that of the parent Schedule P line. Actual IRS loss reserve discounting complicates the modeling, so we use true present values as a proxy.

Illustrations A and B (premiums and losses) derive implied capital flows from underwriting cash flows, reserve requirements, capital requirements, and tax cash flows.

A company writes and collects a $1,000 annual premium on December 31, 20X0, and pays acquisition expenses of $250 on that day. General expenses of $150 are incurred and paid evenly over the policy term. No losses occur on the policy.

The pre-tax investment yield benchmark is 8% per annum (bond equivalent yield). The marginal tax rate on both underwriting income and investment income is 35%.

Capital requirements and costs of holding capital differ by country, depending on insurance regulation and corporate law.

Low: Unpaid losses (including UEPR) are held at fair value, and insurer is domiciled in a country with low capital requirements. Tax haven countries, such as Bermuda, often have low (but not zero) capital requirements.

Medium: Unpaid losses are held at fair value. Capital requirements use the insurer's own ERM model, and they are low for large, well-run firms (as a percentage of written premium or held reserves). Solvency II envisions this structure for European countries.

High: UEPR is the gross premium, so shareholders incur the initial underwriting loss of pre-paid acquisition expenses. Insurers hold full value loss reserves, so shareholders contribute the implicit interest discount. Capital requirements are set by formulas (RBC and rating agency), with the same charge for all insurers. The U.S. has this structure.

The modeling order for any regulatory environment covers five sets of cash flows.

a. underwriting cash flows: premiums, expenses, losses, and taxes on underwriting
b. statutory accounting entries: loss reserves and deferred tax assets
c. required surplus amounts: risk-based capital requirements
d. investment income on the investable assets
e. implied capital flows to and from the equityholders
TAX ON UNDERWRITING INCOME

Current and deferred taxes affect implied capital flows. For the U.S. scenario, taxable underwriting income = written premium – underwriting expenses – 80% of the change in the unearned premium reserve – paid losses – the change in the discounted loss and loss adjustment expense reserves. Taxable underwriting income for 20X0 is

$200 \text{ (income from revenue offset)} - $250 \text{ (acquisition expenses)} = -$50.

The tax liability is 35% times the taxable income: \(35\% \times ($200 - $250) = -$17.50\). This tax refund of $17.50 does not rely on tax carrybacks or carry-forwards. The tax refund offsets tax liabilities from other policies and from investment income.

Taxable underwriting income for 20X1 equals

$800 \text{ of taxable premium income} - $150 \text{ of general expenses} = $650.

The tax liability is $650 \times 35\% = $227.50. Written premium during the year is $0 and the unearned premium reserve declines to $0. Tax basis earned premium is $0 – 80\% \times (-$1,000) – $150 = $650 or $1,000 + 20\% \times (-$1,000) – $150 = $650. The tax on underwriting income is incurred evenly over the year, or $227.50 / 2 = $113.75 each half year.

We model taxes separately for premiums and losses, showing tax liabilities and deferred taxes for each. We assume no systematic risk of premium or losses, so the capitalization rates are the same. We use the insurer’s investment yield and its target return on capital, without separating the return into investment vs underwriting portions. We have not yet included the investment income cash flows, since these depend on statutory accounting constraints and on the capital requirements.

STATUTORY ACCOUNTING ENTRIES

- The unearned premium reserve is amortized as the insurance protection is provided.
- Surplus of $250 is added on 12/31/X0 and removed on 12/31/X1; it is not amortized over the policy term. The required capital is based on risk-based capital charges on the written premium for the year and the loss reserves at the valuation date.
- A deferred tax asset of $200 \times 35\% = $70 from revenue offset is accrued on 12/31/X0 and amortized over the policy term as the unearned premium reserve runs off. The full deferred tax asset from revenue offset is recognized on the statutory balance sheet, since it reverses within 12 months.

IMPLIED CAPITAL FLOW AT POLICY INCEPTION
Implied capital flows affect investment income, and investment income affects the implied capital flows. We model the implied capital flow at policy inception and then the investment income for the first half of the year.\textsuperscript{17}

- The required assets on December 31, 20X0, are $1,000 of unearned premium reserve plus $250 of required surplus, or $1,250.
- The assets from underwriting transactions are $1,000 (written premium) – $250 (acquisition costs) + $17.50 (tax refund) + $70.00 (deferred tax asset) = $837.50.
- The equityholder provided capital is $1,250 – $837.50 = $412.50.

The equityholder provided capital is the equity in the unearned premium reserve plus the supporting capital plus the accrued taxes (current liability minus the deferred tax asset):

\[
$250 \text{ (equity)} + 250 \text{ (surplus)} – (–17.50 – 70) = 412.50
\]

For regulatory environments with principles-based capital requirements depending on the insurer’s own ERM model, the required capital is highest at policy inception and declines as the premium is earned and losses are incurred.

\textit{Illustration}: An insurer’s ERM model requires capital equal to 50% of unpaid losses before they occur and 20% after they occur. If the expected loss ratio is 70% and 25% of losses are paid during the policy term, the required capital declines from 50% × 70% = 35% of written premium at policy inception to \((1 – 25\%) × 70\% × 20\% = 10.50\%\) at expiration.

\textit{Investment Income}

The invested assets on 12/31/20X0 are $750 (net premium) + $412.50 (capital) + $17.50 (tax refund) = $1,180; the deferred tax asset of $70 is not investable. Equivalently, required assets are $1,000 (unearned premium reserve) + $250 (surplus) = $1,250. Subtracting the $70 non-investable deferred tax asset = $1,180 investable assets.

- The investment yield is 8% per annum compounded semiannually.
- The investment income of 4% per period is received on 6/30/20X1 and 12/31/20X1.
- Unearned premiums decline to $500 on 6/30/20X1 and $0 on 12/31/ 20X1.
- The deferred tax asset declines to $35 on 6/30/20X1 and $0 on 12/31/ 20X1.

The investment income during each period is the investable assets at the beginning of the period times the investment yield during the period.\textsuperscript{18}
Illustration: The investment income earned in the first half of 20X1 is 4% × $1,180 = $47.20, received on June 30, 20X1. The assets required on July 1, 20X1, are $500 of unearned premium reserve plus $250 of required capital = $750. The non-investable deferred tax asset is $500 × 20% × 35% = $35, so investable assets are $750 – $35 = $715. The investment income earned in the second half of 20X1 and received on 12/31/0X1, is $715 × 4% = $28.60.

In a fair value environment, if taxable income equals economic income, the deferred tax assets and liabilities are generally zero. If the insurer has no receivables, investable assets equal surplus plus reserves.

Investment Income and Underwriting Expenses

We subtract underwriting expenses to determine the tax liability on underwriting income but not to determine investment income. The investment income is the investment yield times investable assets, which are the statutory reserves and required capital (less any non-investable assets). Expenses are replenished with equityholder supplied funds; they do not affect investable assets. An expense outflow causes a federal income tax inflow (to the insurer) and an implied equity outflow (from investors to the insurer).

- The federal income tax inflow offsets 35% of the incurred expense.
- The implied equity outflow offsets the other (1 – 35%) of the expense.\(^{19}\)
- The net change in the company’s assets is zero: 100% expense outflow – 35% federal income tax inflow – 65% implied capital flow into the company.

In practice, the amortization of the unearned premium reserves is offset by the accrual of loss reserves. To highlight the equity inflows as premium is earned and the equity outflows as losses are incurred, this illustration models premiums and losses separately.

Taxes on Investment Income

Federal income taxes on investment income are (i) $47.20 × 35% = $16.52, paid on June 30, 20X1, and (ii) $28.60 × 35% = $10.01, paid on December 31, 20X1. The tax liability is computed for the year as whole, but payments are made quarterly in advance. For simplicity, we show payments on June 30, 20X1, and December 31, 20X1.\(^{20}\) We use the pre-tax investment yield and model federal income taxes explicitly.\(^{21}\)

Implied Capital Flows

The implied capital flow is positive for a capital distribution (a stockholder dividend or stock repurchase) and negative for a capital contribution. Common stocks are cash equivalents, so a capital gain – even if unrealized – is a capital distribution; shareholders can always sell some shares to produce a virtual dividend payment.\(^{22}\)
Illustration: On January 1, ABC Insurance has 10 million shares outstanding at $50 a share, for a market value of $500 million. The company earns $50 million during the year and has a market value of $550 million on December 31.

- **Scenario A:** If the company pays a dividend of $5 per share on December 31, its market value declines to $500 million. The stockholder dividend is the capital flow.
- **Scenario B:** If the company pays no dividend, its market value remains $550 million. Since common stock can be sold readily, the liquid assets of the owners increase by $5 per share. The capital accumulation is the implied capital flow.\(^{23}\)

We may conceive of the implied capital flow from a balance sheet perspective or from an income statement perspective. Both methods use the accounting entries for the insurance regulation in the domiciliary country, such as NAIC accounting for the U.S.

- **Balance sheet perspective:** At valuation dates, unneeded assets are distributed to equityholders and insufficient assets are augmented by equityholder contributions. At the start of each new period, the held assets equal the required assets, which are the statutory liabilities plus the required capital. During the valuation period, investable assets increase by net cash inflows and non-investable assets change by net non-cash changes. The implied capital flow at any valuation date is the change in held assets minus the change in required assets. Since the held assets equal the required assets at the previous valuation date, this is the held assets minus the required assets.

- **Income statement perspective:** The implied capital flow at valuation date \(t\) equals the statutory income during the period from \(t-1\) to \(t\) minus the change in required capital. A direct charge or credit to surplus is treated as a component of statutory income.

We illustrate the two methods for the implied capital flow on June 30, 20X1.

**Balance sheet perspective:** The assets held on January 1, 20X1, of $1,250 ($1,180 of investable assets plus a $70 deferred tax asset) are the required assets at that date: the unearned premium reserve plus required capital. The investable assets are accumulated for investment income ($47.20) and decreased for expenses ($150) and tax accruals ($113.75 tax accrual for underwriting income; $16.52 tax accrual for investment income).\(^{24}\) The deferred tax asset is amortized for half a year (−$35). The held assets on June 30 are $1,250 + $47.20 − $150.00 − $113.75 − $16.52 − $35.00 = $981.93.

The required assets on June 30 are the statutory unearned premium reserve ($500) + the required capital ($250), or $750. The implied capital flow is $981.93 − $750.00 = $231.93.

**Income statement perspective:** The statutory income during the first half of 20X1 is shown below. Direct charges or credits to surplus, such as the change in the deferred tax asset, are included with statutory income.
There is no change in the required capital, so the implied capital flow is +$231.93.

We calculate the implied capital flows for the second half of the year in the same two manners. For heuristic purposes, we show a third method below. A change in the implied capital flows between the first half of the year and the second half stems from changes in statutory income or changes in capital requirements. We list these differences below.

- General expenses are $150 on 6/30/20X1 and $0 in the second half of the year.25
- Investment income is $47.20 in the first half of the year and $28.60 in the second half. The difference in the investment income reflects the difference in the investable assets.
- The federal income tax on investment income is $16.52 in the first half of the year and $10.01 in the second half of the year.
- Required capital declines to $0 on December 31, 20X1.
- The tax liability and the amortization of the DTA are spread evenly over the year.

We adjust the implied capital flow from June 30, 20X1 with these differences to get the implied capital flow on December 31, 20X1:

\[
\text{Capital flow on June 30, 20X1} = +$231.93 \\
\text{Difference in general expenses} = +$150.00 \\
\text{Difference in investment income} = +($28.60 - $47.20) \\
\text{Difference in federal income taxes on investment income} = -($10.01 - $16.52) \\
\text{Difference in surplus change} = +$250.00 \\
\text{Capital flow on December 31, 20X1} = +$619.84
\]

These adjustments highlight the sources of implied capital flows.26

The accompanying graphic shows the cash flow view of the implied capital flows. The following table shows the income statement view of the implied capital flows.

**ILLUSTRATION B – LOSS TRANSACTIONS**

*Losses occur evenly during the policy term and are paid over several years. To simplify, we model two losses of $400 each occurring on June 30, 20X1, and December 31,*
20X1; both losses are paid on December 31, 20X3.\textsuperscript{27} Tax rates and capital requirements are the same as before.

The additional cash flows are a loss payment on 12/31/20X3 of $800, investment income for two and a half years (6/30/20X1 to 12/31/20X30), and federal income tax payments or refunds over this period. Non-cash changes to the balance sheet are the capitalization and amortization of loss reserves and deferred tax assets from loss reserve discounting. The RBC reserving risk charge increases the capital requirements.

**Capital flow principles**

We divide loss reserves into two pieces:

- The present value of unpaid losses at a risk-free rate with no risk margin.
- The implicit margin held in statutory reserves, called the implicit interest discount.

The fair value risk margin for unpaid losses is the output of the pricing analysis, not an input. It is the cost incurred by the insurer (and paid by policyholders) for underwriting losses, above the present value of losses and expenses.

Modern solvency regulation is replacing implicit margins with explicit margins in surplus. But U.S. statutory accounting still retains the implicit margins in policyholder reserves. The RBC formula subtracts the implicit margin in reserves from total capital requirements to derive surplus requirements. For policy pricing, we add the implicit margin in reserves with the explicit margin in surplus to derive the capital requirements.

**Federal Income Taxes**

We use after-tax cash flows to determine implied equity flows and the internal rate of return or net present value to investors. If tax basis loss reserves are present values at risk-free rates with no risk margin (as in the United States), the tax basis unpaid losses are too low and the tax liability is too high, further increasing the double taxation cost of holding capital.

Pricing studies use IRS loss reserve discount factors by line of business and accident year to compute taxable income and tax liabilities.\textsuperscript{28} This illustration assumes IRS loss reserve discount factors of 86%, 88%, and 90% for accident year 20X1 at 12 months, 24 months, and 36 months.

The tax offset for 20X1 is the tax rate times the change in the discounted reserves. Since losses first occur in 20X1, this is 35% × 86% × $800 = $240.80.\textsuperscript{29} We pro-rate the tax liability among the two halves of the year, or $120.40 at 6/30/X1 and at 12/31/X1.

Similarly, the tax rate times the change in discounted reserves for 20X2 is 35% × (88% × $800 – 86% × $800) = $5.60, or $2.80 for each half year. The tax basis incurred loss in 20X3 is the paid loss plus the change in discounted reserves: $800 + (90% × 0 – 88% ×
$800) = $96.00. The offset to the tax liability on December 31, 20X3, is 35% × ($800 + 90% × $0 – 88% × $800) = $33.60, or $16.80 for each half-year.  

**Cash Flow and Capital flow Patterns**

The illustration shows the occurrence and payment of losses in different years. The company cash flows for incurred losses show the following pattern:

- There are significant cash *inflows* stemming from the offset to taxable income on the dates the losses occur (June 30, 20X1, and December 31, 20X1). The cash inflow from the offset to taxable income precedes the cash outflow from the payment of losses.
- There is a large cash outflow on the date the loss is paid, or December 31, 20X3.
- The unwinding of the IRS loss reserve discount is offset by the investment income on the assets backing the discounted reserves. The investment income is a cash inflow offset by a non-cash outflow of the unwinding of the implicit interest discount in the reserves. The combination of these offsetting elements is a net cash inflow equal to the investment income, no taxable income, no tax liability, and no implied capital flow (except for the deferred tax asset).
- The investment income on capital supporting reserves is not offset by amortization of the interest discount in the reserves. The supporting capital is the capital embedded in full value reserves (i.e., the difference between full value and discounted reserves) plus the capital held in surplus for RBC risk charges.
- Changes in the deferred tax asset are not company cash flows.

The implied capital flows show a different pattern. The losses cause large equity outflows during 20X1 and a modest equity inflow on December 31, 20X3. Small implied equity outflows equal to the amortization of the deferred tax asset occur in between.

We assume the insurer posts full value reserves when losses occur. If the coverage is priced adequately, the policyholder premium provides for the present value of the expected losses plus an amount to fund the cost of holding capital and the associated federal income taxes. Investors fund the expected loss reserve with assets equal to

\[ \text{difference between held reserve and fair value of reserve} + \text{capital requirement for reserving risk} - \text{capital provided by policyholders}. \]

To distinguish the components of the loss reserves and the sources of capital, we conceive of the held reserve as the present value of future loss payments plus the capital embedded in the statutory held reserve. The *capital requirement* is the explicit capital in the surplus account. For this illustration, we assume a capital requirement equal to 15% of held loss reserves. This is consistent with current NAIC risk-based capital requirements.

**Computations**
On June 30, 20X1, the loss reserves are $400 and the required capital is $400 \times 15\% = $60.00. On December 31, 20X1, the reserves increase to $800 and the required capital increases to $800 \times 15\% = $120.00.

The federal income tax liability on December 31, 20X1, resulting from the incurred losses is $-35\% \times 86\% \times $800 = -$240.80. The total deferred tax asset (DTA) is $5.60 + $33.60 = $39.20. This is the deferred tax asset on a GAAP balance sheet, as well as the gross deferred tax asset in column 1 of the statutory balance sheet. Of the $39.20 DTA, only $5.60 reverses within 12 months. This is the admitted portion on the 12/31/20X1 statutory balance sheet.

We assume that the tax on underwriting income is paid (or the offset to taxable income is received) evenly over the year. For the illustration, we assume that half the deferred tax asset is accrued on June 30, 20X1, and the other half is accrued on December 31, 20X1.

The assets needed to fund the incurred losses at June 30, 20X1, are

the held reserves + the capital requirement – the tax refund – the deferred tax asset, or

$$400 + 60 - 120.40 - 2.80 = 336.80.$$  

The policyholder funds are provided by the policy premium. At policy inception, the money is transferred to the unearned premium reserves. Over the course of the year, the money in the unearned premium reserves is transferred to the loss reserves. The equityholder funded capital is a capital infusion at the time of the loss occurrence.

If the policy is adequately priced, the premium covers the fair value of the losses plus a margin for the cost of holding capital and the associated taxes. The equityholders provide the capital embedded in the reserves and the capital explicitly held in statutory surplus, minus the capital provided by the policyholders.

The assets needed to support the incurred losses at December 31, 20X1, are:

$$800 + 120 - 240.80 - 5.60 = 673.60.$$  

We have separated premium transactions from loss transactions in this illustration to highlight the relations among company cash flows and implied capital flows.

- The premium transactions show the funds supplied at policy inception (time 0) by both policyholders and equityholders to support the unearned premium reserve and the initial underwriting loss. At times $t=\frac{1}{2}$ and $t=1$, the funds are returned to equityholders.
- The loss transactions show equityholder supplied funds at times $t=\frac{1}{2}$ and $t=1$ to support the loss reserves. At time 3, the remaining funds are returned to the equityholders.
In practice, loss reserves and paid losses gradually replace unearned premium reserves. The policyholder supplied funds collected at policy inception to support the unearned premium reserve are transferred to support the loss reserves as the premium is earned and the losses are incurred. The profit in the policyholder premium is transferred gradually to the tax authorities to cover the costs of holding capital: double taxation on investment income on capital funds plus the underwriting tax on this profit provision.

In the illustration, there is no change in the undiscounted reserves between December 31, 20X1, and December 31, 20X3. We use this simple scenario to clarify the cash flows and implied capital flows, without having to deal with changing loss reserves and paid losses at each valuation date. In practice, reserves run off gradually. Paid losses and statutory reserves change at each valuation date.

**Deferred Tax Asset: IRS Loss Reserve Discounting**

The deferred tax assets are computed at year-end dates. To clarify the exposition, we begin this sub-section with annual valuation periods, not semi-annual periods. We then turn to semi-annual valuation periods to explain the calculation of the deferred tax assets at the mid-year valuation dates. In each section, we also determine the investable assets.

On December 31, 20X1, the offset to statutory income stemming from the incurred losses is $800. The federal income tax offset that would result if taxable income = statutory income is 35% × $800 = $280.00.

- The actual offset to taxable income on December 31, 20X1, is 86% × $800 = $688.00.
- The offset to the tax liability is 35% × 86% × $800 = $240.80.

The actual tax liability is greater than the tax liability implied by the statutory balance sheet. The difference is $280.00 – $240.80 = $39.20. This difference is recouped by tax refunds in subsequent years. Since statutory accounting recognizes the full value loss reserve on the occurrence date, it should recognize the tax offset on that date as well. Tax accounting accrues the last $39.20 as the interest discount unwinds. GAAP and statutory accounting treat this as a receivable and show it as a deferred tax asset on the balance sheet.

GAAP financial statements recognize the full receivable if the firm expects to collect it. Statutory accounting admits only the portion of the deferred tax asset that is expected to reverse within 12 months of the statement date. To calculate the admitted portion of the deferred tax asset on the December 31, 20X1, statutory balance sheet, we estimate the portion of this deferred tax asset that remains on December 31, 20X2.

- On 12/31/20X2, the offset to taxable income is 88% × $800 = $704.00.
- The offset to the tax liability is 35% × 88% × $800 = $246.40.
- The change in the tax liability from 12/31/20X1 to 12/31/20X2 is $246.40 – $240.80 = $5.60.
<table>
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<tr>
<th>Date</th>
<th>Statutory</th>
<th>Tax</th>
<th>Difference</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/31/20X1</td>
<td>$800</td>
<td>$688</td>
<td>$112</td>
<td>—</td>
</tr>
<tr>
<td>12/31/20X2</td>
<td>$800</td>
<td>$704</td>
<td>$96</td>
<td>$16</td>
</tr>
</tbody>
</table>

The admitted deferred tax asset = 35% × $16 = $5.60.

The expected change in the deferred tax asset from the current valuation date to the valuation date one year hence is the portion of the deferred tax asset that is recognized on the statutory balance sheet.

We calculate the investable assets to determine the expected investment income. We show first the procedure for annual valuation periods, and then extend the computations to semi-annual valuation periods. The investable assets equal the total assets minus the admitted portions of any non-investable assets on the statutory balance sheet, such as deferred tax assets.\(^{34}\) The admitted portion of the DTA is $5.60 at 12/31/20X1 and $33.60 at 12/31/20X2. The investable assets are

- $920.00 – $5.60 = $914.40 during 20X2 and
- $920.00 – $33.60 = $886.40 during 20X3.\(^{35}\)

**Semi-Annual Valuation Periods**

The deferred tax asset at December 31 of year X is accrued evenly over year X and declines to zero over year X+1. During year X+1, a new deferred tax asset is accrued, which declines to zero during year X+2.\(^{36}\) Each deferred tax asset follows a two-year accrual and amortization pattern shaped like a carot (“\(^\wedge\)”).

This pattern assumes that losses are paid evenly during the year. Before a loss is paid, there is a gross (GAAP) deferred tax asset associated with its reserve. The deferred tax asset is admitted on the statutory balance sheet only during the 12 month period prior to its payment.

**Illustration:** The $5.60 deferred tax asset at December 31, 20X1, is accrued evenly over 20X1: $2.80 on June 30, 20X1, and the remainder on December 31, 20X1. It declines to $2.80 at June 30, 20X2, and to $0 by December 31, 20X2.

Similarly, a the $33.60 deferred tax asset at December 31, 20X2, accrues evenly over 20X2 ($16.80 on June 30, 20X2, and $16.80 on December 31, 20X2) and declines to $16.80 at June 30, 20X3, and to $0 on December 31, 20X3. The total deferred tax asset on June 30, 20X2, is $2.80 + $16.80 = $19.60.

This modeling procedure interpolates between the deferred tax assets at the end of the year and the beginning of the year to derive the deferred tax asset at June 30.
The deferred tax asset at June 30, 20X1, is \( \frac{1}{2} \times (0.00 + 5.60) = 2.80 \).
The deferred tax asset at June 30, 20X2, is \( \frac{1}{2} \times (5.60 + 33.60) = 19.60 \). The deferred tax asset at June 30, 20X3, is \( \frac{1}{2} \times (33.60 + 0) = 16.80 \).

Although the change in the deferred tax asset is not a cash flow, the recognized portion of the deferred tax asset is an admitted statutory asset. A change in the admitted deferred tax asset causes an implied capital flow. A decrease in the deferred tax asset increases capital required from equityholders, expected investment income, and associated taxes.

The total assets held by the insurer are $920 throughout 20X2 and 20X3. The investable assets are $914.40 at 12/31/20X1, $917.20 at 6/30/20X2, $886.40 at 12/31/20X2, and $903.20 at 6/30/20X2. The investment income in each period equals the investable assets times the investment yield of 4% per half-year (e.g., $914.40 \times 4\% = 36.58\)

<table>
<thead>
<tr>
<th>Period</th>
<th>Total Assets</th>
<th>Deferred Tax Asset</th>
<th>Investable Assets</th>
<th>Investment Income</th>
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<td>1/1– 6/30/X2</td>
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<tr>
<td>7/1 – 12/31/X3</td>
<td>$920.00</td>
<td>$16.80</td>
<td>$903.20</td>
<td>$36.13</td>
</tr>
</tbody>
</table>

The implied capital flow at each valuation date is the statutory income during the preceding period minus the change in capital requirements from the beginning to the end of the period. The components of statutory income in the first half of 20X2 are as follows:

- Investment income is $36.58.
- The tax on the investment income is 35% \( \times 36.58 = 12.80 \).
- The change in the deferred tax asset is $19.60 – $5.60 = $14.00.
- The tax refund for the 20X2 amortization of the loss reserve is $5.60, or $2.80 each half year.

The total statutory income is $36.58 – $12.80 + $14.00 + 2.80 = $40.58. There is no change in the required capital, so the implied capital flow is +$40.58.
For the first three half years in the table above, the implied capital flows equal the statutory income. On December 31, 20X3, the loss is paid, the loss reserve decreases to $0, and the required capital decreases from $120 to $0. The implied capital flow on December 31, 20X3 is $23.48 – ($0 – $120.00) = $143.48.

**PREMIUMS AND LOSSES**

To complete the modeling of the implied capital flows, we overlay the premium transactions with the loss transactions. On June 30, 20X1, and December 31, 20X1, there are implied capital flows stemming from both premium and loss transactions.

**JUNE 30, 20X1:** The implied capital flow from the earning of premiums is +$231.91. This includes the effects of premium earnings, expense payments, federal income taxes, the takedown of one half of the deferred tax asset from revenue offset, and investment income. There is no change during the year in the capital requirements stemming from the written premium RBC charge.

The implied capital flow stemming from the occurrence of the first loss is –$336.80. This includes the effects of loss accrual, federal income taxes, the deferred tax asset stemming from IRS loss reserve discounting, and capital requirements for held loss reserves. The net implied capital flow on June 30, 20X1, is +$231.93 – $336.80 = –$104.87.

**DECEMBER 31, 20X1:** The implied capital flows are +$619.84 from earned premium and –$336.80 from the second loss. This includes the same items as the implied capital flow on June 30, 20X1; it does not include the effect of investment income on the assets held to support the first loss (incurred on June 30, 20X1). The investable assets supporting the first loss are $400 of loss reserve plus 15% × $400 = $60 of supporting surplus minus $2.80 of deferred tax asset = $457.60. The after-tax investment income on these investable assets is $457.20 × 4% × (1–35%) = $11.89. The net implied capital flow on December 31, 20X1, is +$619.84 – $336.80 + $11.89 = $294.93 (premium flows, loss flows, and after-tax investment income on the assets supporting the loss reserves).

**INTERNAL RATE OF RETURN AND NET PRESENT VALUE**

The internal rate of return is the interest rate that sets the present value of the implied capital flows to zero.38

\[
0 = -412.50 - 104.87/(1+x) + 294.93/(1+x)^2 + 40.57/(1+x)^3 + 40.21/(1+x)^4 + 23.05/(1+x)^5 + 143.48/(1+x)^6.
\]

The solution is \(x = 1.485\%\), which is a 3.0% effective annual rate (1.01485^2 = 1.030).39
The cost of capital is at least equal to the investment yield of 8% per annum. The internal rate of return is less than the cost of capital, so the policy generates an economic loss.

Some regulators think a positive internal rate of return implies a profit, even if the profit is not as great as the company desires. Net present value models, which show a dollar gain or loss, circumvent this error. An IRR less than the cost of equity capital produces a dollar loss, and an IRR more than the cost of equity capital produces a dollar gain.

For performance measurement, insurers use EVA (economic value added) yardsticks in addition to the IRR. Applying a net present value analysis to the implied capital flows (not to the company cash flows) is similar to an economic value added analysis. Both net present value and economic value added translate the implied return into a dollar amount, so that the gain or loss to the company is more readily understood. But NPV and EVA do not adjust for the volume of business. One line of business may have a higher NPV or EVA than another because it is larger, not because it has a higher return on capital. If capital is the major constraint on business volume, IRR is the best performance measure. If capital is available without constraint to write profitable business, NPV and EVA are better performance measures.

These illustrations cover the major capital flows that affect insurance pricing models. Realistic models have more entries, but they are not conceptually different.

Some readers might dismiss this analysis as needless precision – that the modeling of tax cash flows and deferred tax assets and liabilities imposes high costs for little benefit.

This may have been true when computations were done with pencil and paper. There is much arithmetic, but the principles are straight-forward. The task of the pricing actuary is to construct the pricing model and to provide reasonable assumptions for the cash flows; the arithmetic is done by computer.

Time demands on actuaries hinder some insurers from developing financially sound pricing models. They are tempted to use popular short-cuts, such as

- traditional combined ratio targets with discounted loss ratios,
- after-tax investment yields instead of explicit modeling of federal income taxes,
- present values of insurance costs instead of the cost of holding capital.

These short-cuts may lead to severe pricing errors in long-tailed lines of business.
**Pricing the Policy**

When computations were done by pencil and paper, actuaries used algebraic methods for rate indications, deriving an expression for the indicated premium as a function of losses, expenses, and other parameters.

Pricing models now use numerical methods with built-in spread-sheet functions like Excel’s *goal seek* and *solver*. We determine the internal rate of return as a function of the premium and the pricing parameters, and we vary the premium so that the internal rate of return equals the opportunity cost of capital.

The losses divided by the indicated premium gives the target loss ratio. The target loss ratio plus the expense ratio gives the target combined ratio.

- Underwriters use the target loss ratio to decide whether to accept an application.
- Actuaries use the target combined ratio to determine the indicated rate change.

**Inflation**

Actuaries and other investment professionals have long argued about the effects of inflation on property-casualty insurance profitability.

- Many writers assumed that inflation hurts insurers by raising ultimate loss costs, since loss trends are assumed to track inflation.
- Butsic argued that if inflation is correlated with interest rates, higher inflation raises both investment income and ultimate losses, with no net effect on profitability. If some loss reserves are not inflation sensitive, higher inflation may increase profitability.\(^{41}\)

An insurer’s profitability depends on its capital requirements and its cost of holding capital, which derive primarily from the cost of double taxation. Inflation raises capital requirements by increasing the implicit interest discount in the full value loss reserves, and it raises the cost of holding capital, since taxes are based on income in nominal dollars, not in deflated dollars. Higher inflation reduces profitability, even with no effects on fixed income assets.

**Business Decisions**

The business in this illustration is unprofitable as an insurer’s entire portfolio. But suppose an insurer is operating profitably and it is offered this policy as an additional risk.

- The net present value of the additional policy is positive, at a risk-free rate.
- Underwriting risk is not systematic; the insurer’s shareholders are not concerned about the unique risks of underwriting.

If the shareholders contribute additional capital to support the additional policy, one might argue that the new policy is unprofitable. But suppose shareholders reason:
The new policy has a positive net present value and contributes nothing to systematic risk. The costs of bankruptcy are minimal for property-casualty insurers, and since we have limited liability, we gain from additional unique risk. If we do not contribute more capital, the new policy raises the profitability of the insurer. Surely the rating agencies see the higher net income of the insurer, which more than offsets any unique risk added by the new policy. If the new policy is in a different location, such as a different state or a different territory, even the total risk of the insurer is diversified.

Insurers differ in their relations with rating agencies and state regulators. Typically, rating agencies and state regulators set the norms, and insurers abide by them. Required capital is set by formulas, not by theoretical arguments about unique and systematic risk.

The insurer’s business managers, underwriters, and pricing actuaries may desire to write the new risk. But the insurer’s executive management is sensitive to possible downgrades by the rating agencies, and they would veto the new policy.

TAXES

Financial economists take two perspectives on the Tax Reform Act of 1986:

- Some view the new tax law as a prepayment of taxes before they are properly due, so insurers pay more than a 35% rate on pre-tax income.
- Other say the tax refund on the unwinding of the implicit interest discount in the loss reserves is offset by the tax liability on the investment income on the assets backing reserves. The net tax is zero, and insurers pay a 35% rate on pre-tax income.42

The partial recognition of deferred tax assets in 2001 has no effect on this analysis, since the DTA is an accounting adjustment, with no associated cash flow.

This analysis of the 1986 tax law and the 2001 statutory change is flawed. Insurers pay about 60% to 70% of their pre-tax profit to the IRS in the long-tailed commercial lines of business. Taxes affect the cost of holding capital, and any tax change that raises required capital lowers insurance profitability. Much of the cost is shifted to policyholders as higher policy premiums; some of the cost is paid by shareholders as lower stock values.

If insurers held assets backing the present value of reserves and no supporting surplus, the 1986 tax law provides a 35% tax rate on pre-tax income, which uses the present value of reserves and assumes no cost of holding capital. Since insurers hold assets backing the full value loss reserves as well as additional surplus, the 1986 tax law provides a 60% to 70% tax rate on pre-tax income for the long-tailed lines of business. Since underwriting has little systematic risk, most taxes stem from statutory reserve and capital requirements.

Deferred tax assets reduce required capital. For other firms, deferred taxes are generally smaller than the tax liability. For property-casualty insurers, the opposite is often true. If an insurer charges premium equal to the present value of losses and expenses, and if the
acquisition expense ratio is 20%, the tax liability on underwriting is zero, but the deferred tax assets are large.

**CONCLUSION: CONSUMERS’ VALUE AND SUPPLIERS’ COSTS**

Insurance consumers compare their premiums with the benefits they receive.

- Insurance expenses, such as legal fees, acquisition costs, taxes, underwriting costs, and claims handling expenses, are not paid to consumers.
- Claim payments benefit consumers with severe losses; other consumers see only high premiums and no benefits.

Even objective economists may find the aggregate consumers’ premiums too high for the benefits received. We compare two perspectives: the consumers’ value perspective and the supplier’s cost perspective.

**Suppliers:** The *supplier’s cost model* described here determines the price that investors demand to fund the insurance operations. To optimize shareholders’ return on invested capital, an insurer focuses on the invested capital and the return on that capital.

**Consumers:** The *consumer’s value model* focuses on the value received by the consumer from the insurance product. A rational consumer looks at the expected cash flows to and from the insurance company. Evaluating insurance from the consumer’s perspective focuses on expected losses and benefits.

The expected loss costs give the pure premium. By adding underwriting expenses and income taxes on the policy cash flows, and discounting at an appropriate interest rate, one determines the present value of benefits. This is the value of the product to the consumer. The consumer does not include the insurer’s cost of capital in the value of the product.

In theory, the consumer’s cost of capital should be included in the consumer’s value perspective. In practice, few consumers set aside capital to fund insurance risks.

*Illustration:* An employer self-insuring its workers’ compensation exposures faces process risk from random loss fluctuations. In theory, it should hold capital to guard against adverse loss fluctuations and to ensure payment of benefits to injured employees. In practice, the employer pays benefits from its cash flow, since it has no reserve or capital requirements.

The capital costs imposed by state regulation widens the difference between the value to the consumer and the cost to the insurer. Even if the insurer and consumer have the same expected insurance cash flows, they calculate different prices for the insurance product. The common result is an inefficient battle of wits among pricing actuaries, regulators, and consumer representatives at state rate hearings.
The size of the difference between the consumer’s value perspective and the insurer’s cost perspective is not always appreciated by regulatory authorities. This difference was narrow at the beginning of the 20th century, when property-casualty products covered primarily the short-tailed property lines of business and capital requirements were low. The difference has risen steadily through the 20th century, as casualty lines have increased, payment patterns have lengthened, and more stringent capital requirements have been imposed. The capital to assets ratio for property-casualty insurers is now many times higher than for life insurers, commercial banks, or other financial institutions. The costs of this high capital to asset ratio should be understood and properly weighed by state regulators.43

Endnotes:

1. For high deductible business, the IRS discount factors are too high, reducing taxable income; for first dollar coverage, the IRS discount factors are too low, raising taxable income.

2. If deferred tax assets are recognized, the difference between statutory income and taxable income affects investable assets, not implied capital flows. Non-admitted deferred tax assets increase the required capital contributions. This illustration uses the present value of unpaid losses as the tax basis reserve, assuming no deferred tax assets.

3. This paper is a template for the practitioner, not only a research paper for the theoretician. We show the calculations step-by-step, so that readers can replicate the procedures.

The documentation has three parts: exposition, exhibits, and graphics. Some readers may find it helpful to trace the figures in the exhibits and the graphics as they proceed through the text. Other readers may prefer the text alone and find the graphics distracting.

We divide the illustration into two parts: premiums vs losses. The capital, tax, and deferred tax asset aspects differ for the two parts, and a combined illustration is complex.

4. A cash inflow (premium) of $1,000 occurs on December 31, 20X0. If the premium is not fully collected up front, the company has a receivable for the uncollected portion.

- If the receivable is an admitted asset, there is no effect on the implied capital flows.
- A non-admitted premium receivable increases the implied equity outflow.
- Even if premium receivables are admitted, the premium collection pattern affects investment income, which affects the implied capital flows in subsequent periods.

For business with significant deferral of premium collection, such as large account
workers’ compensation policies, the actuary may estimate the expected non-admitted portion of the premiums receivable asset and increase the required capital contribution.

5. GAAP defers most acquisition expenses, with no effect on income. Actuarial ratemaking treats acquisition expenses as part of the policy premium, with no effect on capital flows. Neither approach is proper.

The modeling for premium collection and acquisition expenses depends on the distribution and premium billing systems. Companies use a variety of distribution systems, such as independent agency, direct writing, salaried sales force, and mass marketing.

- Agents’ commissions are incurred on the policy writing date or premium collection date.
- For direct writers and for companies with salaried sales forces, advertising expenses and fixed costs of the agency system are incurred before the policy effective date.
- For commercial lines companies writing large accounts, the costs of developing sales proposals and soliciting business also occur before the policy effective date.

Pricing actuaries who ignore the timing of acquisition expenses under-estimate the required premium, especially if the distribution system requires high capital expenditures at or before policy inception. The illustration here uses commission plus underwriting expenses of $250 paid on the policy effective date. The initial underwriting loss makes the implied capital flows differ from the company cash flows. General expenses of $150 occur evenly over the policy term; we model them as a single $150 outflow on June 30, 20X1.

6. Some actuaries estimate profitability from the present value of all cash flows at a risk-free rate: a positive net present value is profitable and a negative NPV is not. This rule under-estimates required premiums.

Illustration: An insurer collects $1,000 of premium at time 0. It pays $250 of acquisition costs on that date, $100 of maintenance expenses during the policy year, and $800 of losses at time 3. The risk-free rate is 8%.

We assume maintenance expenses occur at time ½. The present value at an 8% capitalization rate is

\[
$1,000 – $250 – $100 \times 1.08^{-\frac{1}{2}} – $800 \times 1.08^{-3} = $18.71
\]

One might argue that the insurer’s capitalization rate is greater than the risk-free rate. But a higher capitalization rate gives a higher net present value. Using 12% per annum gives
$1,000 – $250 – $100 \times 1.12^{\frac{1}{2}} – $800 \times 1.12^{-3} = $86.08

The rule-of-thumb underprices this policy, since it does not consider capital or reserve requirements.

7. As mentioned earlier, the profit margin in the premium is predominately the cost of holding capital grossed up for taxes on underwriting income. Since 1986, actuaries have argued whether federal income taxes are pre-paid and too high or correctly match revenues with expenditures and are just right. Neither side is correct. If underwriting risk is diversifiable, the needed underwriting return is zero and the tax on this return is zero. The actual profit margin in the premium stems predominantly from the cost of holding capital, which reflects the use of nominal interest income, the multiple year effect, and the gross-up for underwriting income. If the premium equals the present value of losses and expenses, the taxes paid by the insurer would still be large. The tax liability is too high, but this has nothing to do with pre-paying taxes.

8. This may also be written as statutory earned premium + 20% of the change in the statutory unearned premium reserve – underwriting losses – statutory incurred losses + the change in the reserve discount.

9. The taxable premium income of $200 may be evaluated either as written premium minus 80% of the change in the unearned premium reserves = $1000 – 80% × $1000 = $200 or as statutory earned premium income plus 20% of the change in the unearned premium reserves = $0 + 20% × $1000 = $200.

10. Alternatively, if all general expenses are assumed to be paid exactly on June 30, 20X1, the underwriting income is $400 – $150 = $250 in the first half of the year and $400 – $0 = $400 in the second half of the year.

11. Myers and Cohn [1987] distinguish between the taxes stemming from premium earning and those stemming from loss accruals because they use different capitalization rates to determine the present values of each. We do not assume any systematic risk for underwriting, and we model all tax cash flows similarly.

12. Some analysts say: Taxes are 35% of pre-tax income, with lower rates for municipal bonds and stock dividends. We determine the profit margin with no taxes and divide by (1–35%). If equityholders need a 5% return on premium with no taxes, they need a 5% / (1 – 35%) = 7.69% profit margin with taxes.

An alternative perspective views the profit margin in the premium as the compensation for the double taxation of the investment income on capital and surplus funds. If the premium to surplus ratio is 1.5 to 1 and the risk-free rate is 6% per annum, the cost of double taxation is 6% × 35% = 2.10% of surplus, or 2.10% / 1.5 = 1.40% of premium.

If there were no taxes, the required profit margin in the premium would be extremely
small. Dividing the pre-tax profit margin by \((1 - 35\%)\) leaves the premium inadequate. The conversion of the double taxation on capital into a margin on premium misses an entire layer of tax on underwriting income. Both methods ignore the compounding effects on multi-year contracts, deferred tax assets from revenue offset and loss reserve discounting, and the statutory limits on recognition of the deferred tax assets.

For long-tailed lines of business like workers' compensation and commercial liability, taxes are 60% to 70% of the profit margin in the premium. Many insurance accountants believe taxes are only 35% of the pre-tax profit margin, since they do not connect the tax liability, which stems from all pre-tax income, to the profit margin in the premium. We can not model insurance operations without careful examination of the tax flows.

13. In the past, some casualty actuaries differentiated between investment income from policyholder supplied funds and investment income from equityholder supplied funds; cf Bailey [1967]. The investment income on policyholder supplied funds depends on the underwriting cash flows; the investment income on equityholder supplied funds depends also on the accounting entries and the capital requirements. The rationale was that policyholders were entitled to the investment income on their own funds but they were not entitled to the investment income on capital and surplus funds.

Although this distinction is not relevant to return on capital pricing models, it is useful for modeling the source of profits. If premiums are exactly adequate, the profit in the policyholder supplied funds (sometimes called policyholder supplied capital) is needed to fund the difference between the cost of equity capital and the after-tax investment yield on equityholder supplied capital. Myers and Cohn [1987] have a similar perspective, though they fail to take into account the equityholder supplied capital embedded in the statutory loss reserves and the gross unearned premium reserves.

14. In theory, new business underwriting risk is highest at policy inception and declines linearly to zero at policy expiration. One might model the capital requirement as twice the statutory required capital for new business underwriting risk when the policy is written, declining linearly to zero over the policy term. The level capital requirement in the text gives the same policy premium and is simpler to model.

15. Atkinson and Dallas [2000], chapter 8, determine the risk-based capital requirements at year-end dates and they discount at the after-tax investment yield to the beginning of the year.


17. We discuss the implied capital flows in more detail further below.
18. Some actuaries use the average investable assets during the period and a continuously compounded investment yield.

19. The implied equity outflow is an investment in the insurance project, so it is modeled as a negative number. The equity outflow is an inflow to the company. Note that the premium is a pre-tax cash flow, so there is an offsetting tax liability or return. A capital flow is an after-tax flow, so there is no offsetting tax liability or return.

20. The actual payment date is earlier, but the premium is also earned earlier, since it is earned evenly over the policy term. As long as the timing of the premium earning and the federal income taxes are consistent, the model is not materially biased.

Some pricing models assume that tax liabilities for the current year are paid evenly during the year. The total investment income for the year is $47.20 + $28.60 = $75.80 and the tax is $75.80 \times 35\% = $26.53. If we assume that taxes are paid evenly between the two halves of the year, the tax payments at each semiannual valuation date are $26.53 \div 2 = $13.265.

The IRS allows taxpayers some leeway in the quarterly tax estimates, and companies differ in the timing of their tax payments. Modeling the precise tax payment stream is complex. It does not have a material effect on the rate indications, as long as reasonable assumptions are used.

The procedure in the text of the paper splits the tax on underwriting income evenly between the two halves of the year, but it computes the tax on investment income based on the investment income earned in each half of the year. It is difficult to quantify the amortization of the loss reserve discount between the two halves of the year, so we use an even spread. In contrast, using an after-tax investment yield is not difficult.

21. Some actuaries use an after-tax investment yield (cf Atkinson and Dallas [2000], chapter 8). If done correctly, this is fine, but it is hard to do correctly. Some tax on investment income is offset by interest unwinding on the loss reserves, and taxes on underwriting income create deferred tax assets or liabilities.

22. The choice between paying dividends and allowing the capital to accumulate in the company depends on the investment opportunities of the company and the tax situation of the equityholders. This choice affects the personal tax liabilities of the investors and cost of holding capital. In practice, it is difficult to model personal income taxes, since they vary with the tax situation of the equityholders.

23. For the cost of holding capital, stockholder dividends differ from capital accumulation, since the tax liability differs between them. This affects the target return on capital, not the implied capital flows.
24. The tax payments are computed separately for underwriting income and investment income:
   - underwriting income: $227.50 for the full year, or $113.75 for each half year, and
   - investment income: $16.52 for the first half year and $10.01 for the second half year

25. This difference stems from the modeling assumptions. In truth, the maintenance expenses are incurred evenly over the year, but we assume a single payment date to simplify the modeling. For the federal income tax payments on underwriting income, we implicitly assume that the maintenance expenses are incurred evenly over the course of the year, and we spread the income tax on underwriting income evenly over the year.

26. Instead of using values at the end of the period, some actuaries use average values for each entry and implied capital flows in the middle of the valuation period. To do this, we amortize the unearned premium reserve and the deferred tax asset evenly over the policy term. The invested assets decline from $1180 on December 31, 20X0, to $250 on December 31, 20X1. The average invested assets are ($1180 + $250) / 2 = $715.00, and the investment income is $715.00 × 8% = $57.20. This is less than the 20X0 investment income of $47.20 + $28.60 = $75.80 in the illustration. The lower investment income is offset by the earlier implied capital flows in the alternative model, and there is no material change in the model results.

27. The single payment date simplifies the flows yet leaves enough detail to highlight the modeling principles.

28. The text presumes knowledge of IRS tax calculations and the post-codification statutory accounting rules for deferred tax assets and liabilities. For prospective pricing, the actuary estimates future loss reserve discount factors based on federal mid-term rates and either industry or company loss payment patterns; the Treasury’s discount factors have not yet been promulgated.

29. The tax offset may also be calculated as 35% × (statutory incurred losses minus the change in the IRS loss reserve discount), or 35% × {800 – (1 – 86%) × 800} = 35% × {800 – 112} = $240.80.

30. Unless a policy is written on December 31 or January 1, there are two accident years for tax purposes. This complicates the pricing model, so we have chosen a December 31 effective date for the illustration. In practice, some actuaries use an effective date of July 1, which is a proxy for the average effective date for policies written evenly through the year. Two accident years are used to evaluate the underwriting tax effects. This is the standard modeling technique for property-casualty insurance policies.
31. The offset is not exact, since it depends on the accuracy of the loss reserve discount factors. The deferred tax asset causes an implied equity outflow equal to the amortization of the DTA. For clarity, the pricing model treats the federal income taxes on the investment income separately from the tax refund on the amortization of the discount in the IRS loss reserves.

32. Slight adjustments must be made for the admitted portion of the deferred tax asset.

33. The risk-based capital requirements for the long-tailed lines of business (except workers’ compensation) are higher than 15%, and companies hold surplus about twice the risk-based capital requirements. The covariance adjustment in the risk-based capital formula reduces the effective risk charge by about 50%. The 15% factor understates the capital requirements for the long-tailed casualty lines of business and somewhat overstates the capital requirements for workers’ compensation.

The illustration does not explicitly distinguish between the reserving risk charge applied to held reserves (R₁) and the asset risk charges applied to the assets backing the held reserves (R₂ and R₃). Because the marginal effect of a risk charge varies directly with the magnitude of the charges in its risk category, the marginal effect of the asset risk charges is only about 10% to 20% of the marginal effect of the reserving risk charges for the long-tailed lines of business. One may conceive of the 15% capital requirement as a 13% to 14% reserving risk charge and a 1% to 2% asset risk charge.

34. Other non-investable assets are agents’ balances, earned but unbilled premiums, and accrued retrospective premiums.

35. The implication of this reasoning seems to be that if more of the DTA is not admitted, the investable assets increase. This raises the investment income, which lowers the need for underwriting income, thereby causing a smaller rate indication.

In fact, the investable assets increase only because the non-admitted DTA is replaced by equityholder supplied funds, which are investable. This raises the invested capital, and it more than offsets the higher investment income. The net result is to raise the rate indication, not to lower it.

36. Since we are using discrete (semi-annual) functions, not continuous functions, “evenly over the course of the year” means one half on June 30 and the other half on December 31.

37. Direct charge and credit to surplus are treated as income.

38. The table at the end of this paper shows all the cash flows, accounting entries, and implied capital flows.

39. By Descartes’ rule, the maximum number of real solutions to this polynomial equation is equal to the number of sign reversals. Since there is only 1 sign reversal,
the 1.485% solution is unique.

40. The National Council on Compensation Insurance, for example, faced this perception among some state rate regulators in the 1980's, when it filed advisory premium indications for its members.

41. Butsic’s reasoning influenced a generation of actuaries, receiving a Michelbacher prize and a place on the actuarial syllabus. Early critiques noted that Butsic failed to consider the effects of inflation and higher interest rates on the market values of long-term fixed-income securities, and his reasoning was correct only for insurers with short duration assets or for inflationary changes occurring before the insurer invested its premium dollars.

All parties to his dispute failed to properly analyze insurance profitability, though Butsic’s paper was a major impetus to proper consideration of an insurer’s total operations.

42. In a wonderful illustration, Butsic showed that with a pre-tax investment yield of 8% per annum and a loss maturity of 18.6 years, if taxes were paid on statutory incurred losses (not discounted losses) as in the pre-1986 law, the policy could be funded entirely by the tax refunds, with a premium of zero.

43. Cf. Anderson, “Gross Premium Calculations . . .” [1959], page 364: “Thus, the stockholders’ viewpoint determines the minimum premium rate, the policyholders’ viewpoint determines the maximum premium rate, and competitive considerations determine the final premium rate between these limitations. In setting the profit objective, only the stockholders’ viewpoint need be considered; the policyholders’ viewpoint and competitive considerations affect only the realization of this objective.”
ATTACHMENT C: FAIR VALUE OF UNPAID LOSSES

FAIR VALUE OF UNPAID LOSSES

The International Accounting Standards Board (IASB) sets accounting practices in the European Union. It has promulgated fair value standards for both assets and liabilities, which are leading to similar changes in the U.S. and other countries. The Financial Accounting Standards Board (FASB) supports fair value accounting for both assets and liabilities, though the treatment of property-casualty loss reserves remains unsettled. This attachment explains the fundamental issues of fair value accounting.

- How fair value is defined and quantified.
- How fair values are estimated for property-casualty unpaid losses.

ACCOUNTING, BUSINESS ACTIVITY, AND UNDERLYING ECONOMICS

Accounting reflects business activity, and business activity reflects the underlying economics. Insurers consider fair values of losses in decision making, and the accounting boards reflect their activity. Accounting practice is shifting from a GAAP paradigm based on historical costs and income recognition as insurance protection is provided to a fair value paradigm based on market values and business decisions. The evolution of loss reserve discounting shows the influence of the new accounting environment.

Twenty-five years ago, statutory, GAAP, and tax accounting used full value loss reserves. High interest rates of the late 1970’s forced insurers to price their policies at present values. Financial reporting soon followed.

- Tax accounting moved to discounted reserves in 1986, with no risk margins.
- RBC solvency monitoring introduced discounted reserves in 1994, using a fixed discount rate.
- Codification of statutory accounting set risk adjusted maximum discount rates for non-tabular discounts. (fn: The SSAP on Contracts uses a constant 150 basis point margin.)
- The FASB and IASB are now setting principles for fair values of insurance liabilities.

Financial reinsurance

Financial reinsurance is a contract that transfers payment obligations to a reinsurer but does not transfer risk. A reinsurance contract that is 100% retrospectively rated with no maximum premium requires the primary insurer to reimburse the reinsurer for loss payments made. The primary insurer does not transfer any loss uncertainty and incurs expenses of shifting money to and from the reinsurer. Some funds withheld contracts use accounting statements to record the transactions, not actual cash flows. The primary insurer holds the cash nominally paid to the reinsurer, records investment income on this cash, the amounts paid to claimants, and the reimbursements made.

Financial reinsurance provides capital management, not risk transfer. The demand for financial reinsurance reflects the implicit reserve discounting in the U.S. market. Fair value accounting relies on actuarial expertise to value unpaid losses on both unexpired and expired exposures.

Fair value risk margins

Actuarial views on loss reserve risk margins have followed three paths.

- Modern finance bases capitalization rates on systematic risk, not diversifiable risk.
Risk adjusted returns on capital are used for projects of different risk and different costs of capital. Risk adjusted discount rates are derived from leverage ratios, costs of capital, and reserve uncertainty.

Returns on risk adjusted capital provide certainty equivalent risk margins. Fair value risk margins are derived from regulatory and rating agency capital requirements.

Statutory accounting shows an initial underwriting loss at policy inception from the gross UEPR and the full value loss reserves, with earnings from investment income as claims run off offsetting the initial losses. Current GAAP shows a more even underwriting loss over the policy term, with earnings from investment income as claims run off. Fair value accounting shows all expected gains and losses at policy inception, reflecting the return for underwriting the policy. Subsequent gains depend on the release of risk margins.
Accounting values: historical cost vs market prices

Accounting systems value assets and liabilities by adjusted cost or market prices. Adjusted cost is the price paid to acquire the asset or transfer the liability adjusted for amortization or depreciation. Property bought for $800,000 on 12/31/2000 with an assumed forty year life has a depreciated value of $600,000 on 12/31/2010. The market value may have changed over these years, and the amortized value does not adjust for inflation, so $600,000 may not be the market value.

Adjusted cost vs market price each have advantages and drawbacks. Relevance, reliability, transparency, comparability, and consistency differ for the two valuation standards. Consider items not traded in active markets but with adjusted costs, such as undeveloped land and unearned premium reserves. GAAP (general accounting) is geared to investors and creditors, for whom market value is most relevant. The cost of land is not a good estimate of its current value if the population of a town changes, and the premium charged for a policy may be a poor estimate of future losses if claim frequency changes. Market value is more relevant for investors and creditors, even if the market value must be estimated by valuation methods.

An accounting value is reliable if different persons get the same value. Adjusted cost is an objective formula, whereas estimates of market values for assets or liabilities that are not traded are often subjective. Adjusted cost is transparent in that users know how it is formed. Estimates of market value from real estate appraisals or actuarial reserve indications are often black boxes to financial statement users.

Quantification of fair value

The fair value of an asset or liability is the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date (SFAS 157, ¶ 5). Fair values are determined by a hierarchy of methods (SFAS 157, ¶ 22):

- **Level 1:** market values for identical assets traded in active (liquid) markets
- **Level 2:** market values for items other than the asset or liability in question.
- **Level 3:** valuation methods for assets and liabilities using unobservable inputs.

SFAS 157 defines the three levels by the type of input data: observable vs unobservable.

- **Observable inputs** are directly seen in active markets.
  - **Level 1** values use market prices for the asset or liability in question.
  - **Level 2** values use market prices of proxies or other inputs. For example, the fair value of a privately placed bond may use market prices of similar publicly traded bonds or risk adjusted discount rates observed in the bond markets.
- **Unobservable inputs** (based on the reporting entity’s information) are used in valuation methods (Level 3). For fair value loss reserves, one might use probability distributions and utility functions (for risk aversion) based on actuarial analyses, not market values.

Regardless of the valuation method, fair values use assumptions of market participants, or unaffiliated parties buying and selling the assets and liabilities. They use private information about the characteristics of the assets or liabilities, but public assumptions for present values. The illustrations below show how this affects fair values of loss reserves.

**Illustrations:** An insurer owns land on the outskirts of an expanding city. It believes the land is highly valuable, based on an internal study of land values around expanding cities, but the market price for the land is low. The fair value of the land depends on the market price, not on the insurer’s internal study. Similarly, an insurer that is highly risk averse may be willing to trade a risky loss reserve for a fixed cash flow 15% higher. If other insurers are less risk averse, the fair value of the loss reserve depends on the average risk aversion. The fair value depends on market valuations, even if the insurer holding the liability would value it differently.
The fair value hierarchy sets an order. Market values are used if they exist; otherwise, proxies are used. Valuation methods are used if no market values or proxies exist.

**Market values:** For assets traded in a liquid market, such as common stocks and corporate bonds, the fair value is the market value. Even if an insurer believes the market over- or under-states the value of a stock or bond, the market value is the fair value. For example, in 2006, many mortgage-backed securities had triple-A ratings and low yields, but over 90% of these securities had fallen below investment grade by 2010. A prescient insurer that foresaw the downgrades would still value the securities at market in 2006, since it could sell the securities at their market values.

Fair values are market based, not entity specific. An insurer may not use its own risk aversion or its own assessment of true value if these differ from market averages. The fair value is an attribute of the asset or liability, regardless of the entity owning the asset or liability, except for credit risk (see below).

**Proxies:** Some assets are not publicly traded. If similar assets are traded in a liquid market, they may be used as proxies. Private equity, privately placed bonds, real estate, executive options, and many assets in emerging economies have no liquid market values, but many have traded proxies. Similarly, some inputs to valuation methods are market based, such as discount rates for bonds of specific credit quality.

**Illustration:** A privately placed bond is not publicly traded, but bonds with the same maturity, coupons, and credit rating are traded. Suppose ABC Corp issues a 20 year privately placed bond with 8% coupons and XYZ Corp, with the same credit rating, issues a 20 year 8% publicly traded bond. One year later, XYZ’s bond trades at 97.50 because of widening credit spreads in the U.S. bond market. We assume the fair value of ABC’s bond is also 97.50, adjusted perhaps for differences in liquidity.

Loss reserves are have no publicly traded proxies. Even the limited public markets (loss commutations and retroactive reinsurance) are not proxies, since reserves are unique. A loss commutation for one set of claims says little about the fair value of other claims.

**Valuation methods**

Valuation methods are used for assets or liabilities that are not publicly traded in active (liquid) markets and have no proxies. Valuation methods give market values if a liquid market exists, and we validate valuation methods using traded securities. Loss reserves are not traded in active markets, so they are valued with valuation methods. Discounted cash flow methods are used for reserves, but no consensus exists on the capitalization rate or risk margins.

**Illustration:** Discounted cash flows give market values of publicly traded bonds, so we use similar DCF methods for fixed-income securities that are not publicly traded. The Black-Scholes formula gives market values for publicly traded financial options, so we use similar formulas for executive options that are not publicly traded.

**Why reserves have no active markets**

Participants in active markets are independent, fully knowledgeable parties who are willing but not compelled to trade (SFAS 157 ¶ 10). Reserves depend on the circumstances of the claim. The insurer against whom the claim is brought knows the likelihood for the claim’s success. Other parties do not know the circumstances of the claim or the likely payments.

Insurer do not trade reserves in liquid markets with other parties having equal knowledge of the claim’s value. The only market prices for reserves stem from retroactive reinsurance and loss commutations. But this information is proprietary and is distorted by numerous items. Loss commutations are often stimulated by financial distress of the reinsurer. The primary insurer may fear that it will receive no reinsurance recoverable
if the reinsurer becomes insolvent, and it accepts a low price for the commutation. The low commutation price does not imply a high capitalization rate or a negative risk margin.

Three items affect the fair value of loss reserves: credit risk, capitalization rates, and capital requirements. Each requires explicit treatment in discounted cash flow models.

Credit risk: Bonds values are reduced for expected non-payments. If the risk-free rate is 8% and a bond issuer should pay $1,100 in a year but has a 1% probability of default with no payment, the fair value of the bond is $99% × $1,100 / 1.08 = $1,008.33. The traditional actuarial view is that loss reserves are not reduced for the likelihood of non-payment. If the risk-free rate is 8% and the insurer must pay $1,080 in one year, its liability is not reduced because it might become insolvent and not pay. The discounted reserve obligation is $1,000, even if it has a 2% chance of becoming insolvent and paying nothing.

SFAS 157 explicitly rejects this distinction. The SFAS explains: “A fair value measurement for a liability reflects its nonperformance risk ... Because nonperformance risk includes the reporting entity’s credit risk, the reporting entity should consider the effect of its credit risk on the fair value of the liability.” (See below for the application to loss reserves.) Liabilities are the mirror image of assets. If a bond is worth $1,008.33 to bondholders, the issuing firm should have a fair value debt of $1,008.33. If the fair value of an unpaid loss to the claimant is $1,000, based on the likelihood of payment by the insurer, the fair value to the insurer is also $1,000.

Capitalization rate: The capitalization rate for corporate bonds is higher than the risk-free rate, even after adjustment for expected defaults. If the risk-free rate is 8%, a firm with a 1% chance of default may issue a one year 10% bond at par. The implied capitalization rate Z is

\[
99% × $1,100 \div (1 + Z) = $1,000 \Rightarrow Z = 99% × $1,100 \div $1,000 − 1 = 8.90%.
\]

The capitalization rate for property-casualty loss reserves is unknown. If it were more than the risk-free rate, the fair value of loss reserves would be less than their discounted value at a risk-free rate, implying that risky reserves are less expensive than risk-free liabilities (ignoring the credit risk of the insurer). Few actuaries think risk reduces the fair value of loss reserves. Most presume that risk raises their fair values, but do not agree on the size of the increase.

Capital requirements: Investors need hold no capital to support stock or bond portfolios, so they have no cost of holding capital. Insurers hold capital to support their loss reserves, and the cost of holding this capital affects the fair value of the reserves. We explain the valuation method, though the capital needed to support loss reserves or the cost of holding this capital are not easily determined.

Market inferences: liabilities = assets – capital

Some actuaries infer the fair value of loss reserves from market values of assets and equity. This method does not value specific reserves and is not used by insurers. But it was an early assessment of the fair value of loss reserves, and its faults highlight the valuation issues.

Suppose an insurer has $100 million of statutory surplus and a 4 to 1 reserves to surplus ratio, so it has $400 million of reserves and $500 million of assets. Assets are corporate and municipal bonds with an average CAPM beta of 0.15. The insurer’s equity has a beta of 0.85.

We infer the CAPM beta of the insurer’s liabilities. If the overall market rises 1%, the assets increase $500 million × 0.15 × 1% = $750,000, and the equity rises $100 million × 0.85 × 1% = $850,000, implying that liabilities decrease $100,000. The implied beta of the liabilities is $–100,000 / (1% × $400 million) = −0.025.

The capitalization rate for the insurer’s liabilities (loss reserves) is the risk-free rate plus beta times the market risk premium. If the risk-free rate is 7% and the market risk premium is 8%, the capitalization rate is 6.8%.
The financial theory underlying this method relies on the presumed ability of CAPM betas to predict market movements. But empirical tests of the CAPM indicate that they have little (if any) predictive ability for market values. Even if stock betas have some validity, the betas of corporate and municipal bonds have little theoretical justification, and the beta of an insurer’s own equity is just a guess. A 95% confidence interval for the beta of loss reserves using this method is so wide that it encompasses all reasonable estimates.

**Present Value Methods**

We determine the fair value of loss reserves with discounted cash flow techniques. Just as the market values of bonds are the present values of future coupon and principal inflows, the fair value of loss reserves is the present value of future loss outflows.

Bonds and loss reserves differ. The discount rates for bonds vary by the maturity and quality of the cash flows. We infer discount rates by bootstrapping from market values. The inferred discount rates are observed values and we validate the valuation method with market values. The result is a Level 2 fair value for non-traded bonds.

The valuation method relies on the consistency in the inferred discount rates for bonds. For bonds of a given type, such as Treasury bonds or high grade corporate bonds, the discount rates derived from one set of bonds give the market values for other bonds. Given market prices for some bonds, we derive fair values for other bonds with similar attributes.

For loss reserves, we do not have market values to validate discount rates, risk margins, or fair values. Actuaries extend bond valuation methods to loss reserves, though differences between bonds and reserves complicate the procedure: stated vs expected cash flows, inflows vs outflows, capital requirements, market verification, and risk margins.

Bonds have stated cash flows: the coupons and principal payments. Expected cash flows of risky bonds differ from stated cash flows because of defaults, which depend on the financial strength of the issuing firm and covenants in the bond indenture. Bond values are the present values of the stated cash flows at a risk adjusted rate that considers expected defaults, uncertainty in the defaults, and constraints on some institutional investors to hold only investment grade bonds. The risk adjusted rate is an observable input. It is not inferred from expected defaults or the volatility of those defaults.

Loss reserves do not have stated cash flows. Uncertainty in cash flows reflects court awards, mortality rates, future inflation, legal rules, and social expectations. Actuaries estimate expected cash flows by projecting historical patterns, adjusted for changes in the insurance and financial environments. They form reserve discount rates from risk-free rates adjusted for these uncertainties.

**Cash Inflows vs Outflows**

The IASB and FASB relate fair values to certainty equivalent cash flows. The fair value of a non-risky cash flow is its present value at a risk-free rate. Most persons are assumed to be risk averse: an uncertain cash inflow has a lower certainty equivalent and an uncertain cash outflow has a higher certainty equivalent. The common perspective is as follows.

Suppose the risk-free rate is 10% per annum and a person will receive $1,000 one year from now. Cash of $1,000 / 1.10 = $909.09 invested at 10% per annum accumulates to $1,000 in one year. The person should be indifferent to receiving $909.09 now or $1,000 in one year. Similarly, a person who must pay $1,000 in one year should be indifferent to paying $909.09 now and letting it accumulate to $1,000 at the risk-free rate.

But suppose the person will receive either $2,000 or $0 in one year with a 50% likelihood of each. The expected cash flow is still $1,000 in one year, but this uncertain cash flow is worth less to a risk averse person.
A risk averse person may accept less than $909.09 to avoid the uncertainty in the future cash flow. If the person is indifferent between receiving $870 now or the uncertain cash flow in one year, the implied discount rate is $1,000 / $870 – 1 = 14.94%.

For a risky cash outflow (payment) of either $2,000 or $0 in one year with a 50% likelihood of each, a risk averse person may be willing to pay more than $909.09 to avoid the uncertainty. If the person is indifferent between paying $930 now or the uncertain cash flow in one year, the implied discount rate is $1,000 / $930 – 1 = 7.53%.

This reasoning implies that risk adjusted discount rates are higher than the risk-free rate for cash inflows, such as common stock returns, and lower than the risk-free rate for cash outflows, such as loss payments.

The financial perspective on risk is that fair value is an attribute of the asset or liability, not of the person receiving or paying the cash. If one person is receiving the cash, another is paying the cash. Bonds are cash inflows to investors and cash outflows from issuing firms. Any transaction has both a buyer and a seller, either of whom may be more risk averse. If ABC buys a bond from XYZ, the certainty equivalent cash flow as outlined above is lower than the expected cash flow for ABC and higher for XYZ.

Market prices do not depend on the perspective from which the transaction is viewed. Modern financial theory explains that only systematic risk affects market prices. Systematic risk is risk that can not be eliminated by diversification. Arbitrage arguments support this perspective. If unique risk affected market value, an arbitrager could buy 50 common stocks with high unique risk at low prices, combine them into a mutual fund with low unique risk (by diversification), and sell the fund shares at high prices. This gain could not persist in efficient markets, since the forces of supply and demand would force prices to converge.

The CAPM relates systematic risk to the covariance of bond returns with market returns, not to the perspective of the investors. Fair value is the market value, which does not differ for the two persons. But the empirical evidence for the CAPM is weak for stocks and nil for bonds. We can explain why only systematic risk should affect fair value, but we can not validate the relation with actual market prices.

The IASB and FASB cite the financial perspective that only systematic risk affects market values. But they view fair value from the perspective of the reporting entity. An uncertain loss reserve with an expected present value of $100,000 might have a fair value more than $100,000 to the insurer and less than $100,000 to the claimant.

DIVERSIFICATION

Investors eliminate unique risks of specific securities by diversification. Insurers with large reserve portfolios diversify risks unique to specific cases but not general reserving risks like mortality improvement or medical inflation. The FASB and IASB have ambiguous comments about systematic vs unique risk.

- They note that finance distinguishes specific risk from systematic risk, but they do not require that only systematic risk be considered or even specify any risk theory.
- They relate fair values to the perspective of the reporting entity. An uncertain loss reserve with an expected present value of $100,000 might have a fair value more than $100,000 to the insurer and less than $100,000 to the claimant.

VALUATION METHODS AND LEVELS

Bond use Level 1, 2, or 3 values, depending on markets and available proxies.

- Bonds traded in active markets use Level 1 market values, not valuation methods. Most corporate bonds and almost all government bonds are publicly traded.
Non-traded bonds with proxies, such as most privately placed bonds, use the discount rate adjustment technique, a Level 2 measure.

Some unusual structured securities have no publicly traded equivalents and use expected present value techniques, a Level 3 measure.

All loss reserves use expected present value techniques, a Level 3 measure.

**Present Value Methods**

SFAS 157 provides three present value methods for assets and liabilities that are not traded in active markets. The methods distinguish stated cash flows (or promised cash flows) from expected cash flows and expected defaults from payment volatility. We explain the methods for bonds and then apply them to loss reserves.

Stated cash flows are listed in the bond’s indenture, assuming no defaults, pre-payments, or delays. (If no cash flows are stated, the most likely cash flows are used instead.) They are observable inputs, not probability distributions that rely on unobservable assumptions. Expected cash flows are unobservable inputs derived from probability distributions of possible cash flows, reflecting defaults, pre-payments, and delays.

The discount rate adjustment technique is the most common valuation method for risky bonds that are not traded in active markets. This technique is used when (i) the bond’s risk is known and (ii) bonds of similar risk are actively traded. This valuation method uses observable inputs (interest rates, yield curves, and issuer ratings), so it is a Level 2 method. It is not used to value loss reserves, which have no observable inputs or proxies traded in active markets.

The bond’s risk depends on the issuer’s financial solidity, the bond’s maturity, and covenants in the bond indenture. Rating agencies (Standard and Poor’s, Moody’s, Fitch) provide credit ratings for publicly traded corporate, municipal, and sovereign bonds.

To value a non-traded bond by this method, we deduce the risk adjusted discount rate from traded bonds of similar risk. The discount rate adjusts for both expected defaults and payment volatility. We apply the rate to the stated cash flows of the bond in question. We do not determine cash flow probabilities or expected values.

The inputs are observable: discount rates inferred from market values of traded bonds and credit ratings of both traded and non-traded bonds. This is a Level 2 valuation method.

**Risk margins**

The type of fair value risk margin depends on the type of cash flows.

- Risk margins for the discount rate adjustment technique cover (i) expected defaults from stated cash flows and (ii) volatility. If the bond is risky, the expected cash flows are less than the stated cash flow. The risk margin compensates for both the expected loss and the uncertainty in the actual cash flows.

- Expected cash flows in the expected present value technique adjust for defaults, so risk margins cover uncertainty only, or the volatility in the actual cash flows.

Stated cash flows differ from expected cash flows, and the two risk margins differ. The fair values from the two techniques should be the same. We estimate the risk margin for the expected present value technique (unobservable inputs) from the risk margin for the discount rate adjustment technique (observable inputs).

**Illustration:** One year Treasury bills trade at 95. One year loans to B- rated firms with 7% probabilities of default have 16% yields. Firm ABC, which was rated B+, borrowed $1,000 with a 12% coupon rate and a two year maturity on January 1, 20X1. One year later, the firm is in financial distress and has a 7% probability of default. Assume the bonds pay nothing on default. (In practice, we include the estimated payment on default in the computations. The assumption in the text is for simplicity only.)
The discount rate adjustment technique uses the observed market yield for firms rated B-. Stated cash flows of $1,120 in one year have a present value of $1,120 / 1.160 = $965.52.

The expected present value technique begins with the expected cash flow for bond ABC of $1,120 × 93% = $1,042 in one year. At a risk-free rate, the present value is $1,042 × 95 / 100 = $969.06. We add a risk margin for payment volatility.

The risk margin may be a certainty equivalent dollar amount or a risk adjusted discount rate.

Certainty equivalent method: If investors are indifferent between the risky bond and a risk-free bond paying $1,040 in one year, the fair value of the risky bond is $1,040 × 95 / 100 = $967.20.

LOSS RESERVES AND EXPECTED PRESENT VALUE TECHNIQUES

Expected present value techniques are used (i) for risky assets with no proxies traded on active markets and (ii) for risky liabilities such as loss reserves. The techniques are Level 3 methods, based on pricing theories and unobservable inputs. Assets using this fair value valuation method are those not traded in active markets, such as:

- A financial option whose cash flows depend on interest rates or prices of other securities.
- A receivable from policyholders whose cash flows depend on economic conditions.
- A reinsurance recoverable that may be uncollectible if the reinsurer incurs other liabilities.

Expected present value techniques use either certainty equivalent cash flows or risk adjusted discount rates. The certainty equivalent cash flow method substitutes risk-free payments for risky payments such that market participants are indifferent between them.

The present value of the difference between the expected cash flows and certainty equivalent cash flows is the fair value risk margin. We assume that insurers, like investors, are risk averse, and prefer non-risky liabilities to risky ones. They prefer a known liability of $Z to a risky one with an expected cash flow of $Z. The risky liability is equivalent to a known liability of $Z + $M, where $M is the fair value risk margin.

The risk margin is an accounting entry that reflects economic preferences. It affects the timing of income recognition and the presentation of the insurer’s results, not the overall cash flows.

Illustration: An insurer collects premium of $1,000 on 1/1/20X1 and expects to pay a $1,100 loss on 12/31/20X2. The risk-free rate is 8% per annum and the risk adjusted loss reserve discount rate is 6% per annum. The insurer posts a $1,100 / 1.06^2 = $979.00 liability for unpaid losses on 1/1/20X1 and reports a $21 profit. This profit is part of the capital supporting the policy. The insurer's pre-tax income from this policy in 20X1 is $979.00 × (1.08 – 1.06) = $19.58. This income is the 2% risk margin released at the end of 20X1. This profit pays for taxes on investment and underwriting income and for cost of equity capital in excess of the after-tax investment yield.

Traditional GAAP and statutory accounting shows the underwriting and investment income stemming from insurance transaction funds by line of business. The investment income from capital and surplus funds is assigned to lines of business if surplus is allocated by line.

Holding capital is expensive, because the after-tax investment income on capital is less than the required return on capital. This cost of holding capital is not shown explicitly in financial statements. To measure total profit, this cost is allocated to line of business, SBU, and policy. The fair value risk margin is the portion of this cost allocated to the block of business.

Statutory accounting shows an initial underwriting loss from expenses incurred when the policy is written. These expenses reduce policyholders’ surplus twice: once as part of the gross unearned premium reserve and once as expenses incurred. The initial underwriting loss is recouped over the policy term. For long-tailed...
lines of business, statutory full value loss reserves causes a further loss during the policy term, which is offset by investment income as the claims are paid.

GAAP does not have the initial underwriting loss from prepaid expenses, which are recorded as a deferred policy acquisition costs asset, not as expenses written off through the income statement. But GAAP has the same mis-match of underwriting losses and investment gains.

Fair value accounting adds a risk margin to the unpaid losses at policy inception. In perfectly competitive markets, if the policy is adequately priced, the initial risk margin reflects the entry price, or the difference between the net premium (after expenses) and the discounted losses.

Actual fair value unpaid losses are exit prices, not entry prices. If premium rates are redundant or deficient, as occurs through an underwriting cycle, exit prices for unpaid losses are below or above the entry prices. Once losses emerge, the fair value unpaid losses are estimated by valuation methods and differ from initial estimates.

Fair value financial statements show the expected profit at policy inception and changes as claims occur and settle. In contrast, GAAP and statutory financial statements show losses during the policy term followed by profits as claims settle.

Traditional accounting shows book profits, or the insurer’s net after-tax accounting income plus or minus direct charges and credits to surplus (or equity). Economic profit, sometimes termed economic value added, is accounting profit minus the required return on capital. The required return on capital and the allocation of capital to block of business are not shown, so users can not measure economic profit.

Fair value accounting uses required returns on capital to set risk margins. The distinction between accounting profit and economic profit is the present value of growth opportunities (the insurer’s franchise value). Premium minus expenses, taxes, and fair value losses is the expected economic profit.

**FAIR VALUE OF A RISKY LOSS RESERVE**

Risky loss reserves must be supported by capital, which is costly to acquire and hold. The insurer incurs two costs: expected losses to claimants and the cost of holding capital to support risky reserves. The latter cost is the certainty equivalent risk margin.

The fair value unpaid losses are the price an insurer must pay a reinsurer in a perfectly liquid market to assume the unpaid losses. This price is the expected losses, the cost of holding capital to support the reserves, and transaction costs (expenses and profit of the reinsurer).

Transaction costs are attributes of the parties not of the losses. In perfectly liquid markets, transaction costs are zero. The IASB and FASB stress that transaction costs do not affect fair values of assets or liabilities.

_Example:_ In one year, an insurer will pay a medical malpractice claim with an expected value of $100,000. It holds $30,000 of capital to support the medical malpractice loss reserve at a cost of 4% per annum. The cost of holding capital is $30,000 × 4% = $1,200. It would be willing to pay a reinsurer $101,200 to assume the risky loss reserve. Its certainty equivalent risk margin is $1,200, or 1.2% of the loss reserve. The present value of the $1,200 at the after-tax risk-free rate is the fair value risk margin at the valuation date. The fair value risk margin depends on the required capital, cost of holding capital, and after-tax discount rate.

Required capital depends on regulators, rating agencies, and economic capital models. All insurers must meet regulatory capital requirements, and most insurers feel compelled to hold additional capital to meet stricter rating agency standards. Some countries are turning to principles-based capital requirements as used in the European Union Solvency II directives, and insurers model the required capital for a given value at risk, tail value at risk, or EPD.
The cost of holding capital is the cost of obtaining capital minus its after-tax investment yield. This cost is greater in countries with high corporate tax rates or high financial friction costs.

Capital is an after-tax cash flow, whose present value depends on after-tax discount rates. The present value of pre-tax cash flows uses pre-tax discount rates, and the present value of after-tax cash flows uses after-tax discount rates. Underwriting risks are unique risks, so the common practice is to use an after-tax risk-free discount rate.

*Excess capital and fair values*

The fair value risk margin depends on market values, not entity-specific values. The certainty equivalent cash flows are those for the average insurer, not for the reporting entity. An insurer who holds excess capital does not have a higher certainty equivalent cash flow.

*Illustration:* If an insurer holds $40,000 of capital for $100,000 of loss reserves, when firms of similar quality hold $30,000, its cost of holding capital uses $30,000, not $40,000.

This technique uses a mix of observable and unobservable inputs. The required capital per dollar of loss reserves and the cost of holding capital are Level 3 unobservable inputs. The reporting entity estimates these values based on actuarial and financial analyses. No values are evident from public markets. The after-tax risk-free rate used for the present value of the cost of holding capital is a Level 2 observable input. Tax rates are set by statute, and risk-free rates may be inferred from market values of Treasury securities or from LIBOR yield curves. The valuation method is Level 3, which is the lowest level of input.

A fair value risk margin is part of the reserve liability, not surplus. In the illustration above, the reserve liability is $101,200, and the additional capital supporting the reserve is $28,800. A reinsurer would demand $101,200 to assume the reserve.

*Risk adjusted discount rates*

Computing certainty equivalent cash flows for each block of business is tedious. Insurers may set fair value risk margins as adjustments to the discount rate.

If the risk margin is m% of discounted reserves each year and the risk-free rate is r, the risk adjusted discount rate is \((1 + r) / (1 + m\%) = r - m\%\). Full value loss reserves are \(L\), discounted loss reserves are \(L / (1 + r)\), and fair value loss reserves are \(L \times (1 + m\%) / (1 + r)\).

Risk adjusted discount rates are particularly useful when required capital is estimated for the entire insurer or an entire line of business, but the risk varies by policy type or account. The target return on capital may be higher for riskier policies, implying a lower discount rate.

Instead of adjusting required capital for the risk of the business, we adjust required returns. This perspective treats insurance like other industries, where capital is needed to build plants and buy equipment, not to cover risk. Investors demand higher returns for higher risk projects, though measuring risk and relating returns to risk is disputed.

Unpaid losses require supporting capital for two risks: adverse reserve development and asset risks on the funds backing the reserves. We use the average investment portfolio of the property-casualty insurance industry to quantify these asset risks.

*Fair value of casualty loss reserves*
The fair value of a liability is the amount that would be paid in an active market to have another party assume the liability. No active market exists for loss reserves, so valuation methods generate the fair value. Liabilities other than loss reserves and policy reserves for insurers and deposits for banks impose no other obligations on the reporting entity. If a firm has a $10,000 liability with payment in one year and the risk-free rate is 10% per annum, the firm could transfer the liability to a bank. The firm deposits $10,000/1.10, which accumulates to $10,000 in one year, and it authorizes the bank to pay its liability. The bank may charge expense fees for its services as dollar amounts or a lower credited rate. But expenses do not affect fair value because the reporting entity does not incur them if it does not actually transfer the liability.

Loss reserves (and bank deposits) differ in that the insurer (or bank) holds capital to support them. Suppose Insurer ABC has a liability on 12/31/20X1 to pay a $10,000 loss on 12/31/20X2, the risk-free rate is 10% per annum, the tax rate is 35%, the insurer’s cost of capital is 15%, and required capital is 20% of reserves. ABC offers to transfer the unpaid loss to Reinsurer XYZ for $10,000/1.10 = $9,090.91.

Fair value risk margins depend on tax laws and financial regulations affecting the reporting entity, so they are specific to the environment in which the insurer operates. This illustration uses tax laws in the United States whose attributes affecting the cost of holding capital are true for many other countries. First, tax basis incurred losses are at present values with no risk margin. Second, corporate earnings are taxed to shareholders. Third, the cost of holding capital is not a tax deductible expense. Insurers operating in many other countries have similar constraints, though the exact figures differ. Insurers in countries with low tax rates (Bermudas, Cayman Islands, parts of Asia and Eastern Europe) and in countries with poor corporate accounting disclosure (some developing countries in Africa, Middle East, and Latin America) have different fair value risk margins.

If XYZ assumes the $10,000 liability, it must hold capital of $2,000, which costs 15% ($300) to obtain. The capital earns after-tax investment income of 10% × (1 – 35%) × $2,000 = $130, so it costs $170 to hold the capital each year. In one year, the $9,090.91 accumulates to $10,000 and the $2,000 accumulates to $2,130.

To assume the liability, XYZ needs $9,090.91 plus the present value of the $170 cost of holding capital. The IASB and FASB say that after-tax cash flows are discounted at the after-tax risk-free rate, and pre-tax cash flows are discounted at the pre-tax risk-free rate. To grasp the rationale, suppose there were no capital requirements (or there were no cost of holding capital). The tax basis unpaid loss is $10,000/1.1 = $9,090.91, which is an expense. The $9,090.91 payment is a revenue, so the taxable income is zero. Over the next year, the cash earns interest income of $9,090.91 × 10% = $909.09, and the liability changes to $10,000. The incurred loss is $909.09, and the net taxable income is again zero.

In contrast, investment income on the present value of $170 is not offset by incurred losses. If XYZ receives $170/1.10, it earns 10% × $170/1.10 during the year, but it keeps only (1 – 35%) × 10% × $170/1.10. Instead, it needs $170/(1 + 10% × (1 – 35%)) = $159.62, which accumulates (after-tax) to $170.

The fair value of the unpaid loss would be $9,090.91 + $159.62 = $9,250.53 if the $159.62 cost of holding capital were a tax deductible expense. But the tax authorities view the fair value as $9,090.91; the $159.62 is profit. Reinsurer XYZ pays tax of $159.62 × 35% = $55.87 on this profit. It remains with $9,250.53 – $55.87, which is not sufficient to pay the loss and the cost of holding capital.

Instead, Reinsurer XYZ needs $159.62 after-tax. ABC $9,090.91 + $159.62 / (1–t), where t is the tax rate of 35%. XYZ has a profit of $159.62 / (1–t), on which it pays tax of $159.62 × t / (1–t). After paying taxes, it has $159.62 (1–t) / (1–t) = $159.62. For t = 35%, $159.62 / (1–t) = $245.57. The fair value of the unpaid loss is $9,090.91 + $245.57 = $9,336.48.

If the unpaid loss will be paid in three years (or N years), not in one year, the procedure is similar. The present value of the loss is $10,000 / 1.1^3 = $7,513.15. In theory, ABC could pay XYZ $245.57 at the beginning of each year for three years to cover the costs of holding capital. The fair value is the present value sum of the cash flows. The $245.57 is not recognized as a tax expense, since capital is an after-tax cash flow. Each $245.57 is taxed as underwriting profit, leaving $159.62. For the first year, the $159.62 plus its after-tax investment
income accumulates to $170: enough to pay the costs of holding capital. The risk margin for the second year
is discounted for another year at the after-tax risk-free rate. The total risk margin is $170 × \sum [(1 + 10\% × (1–

The risk margin accords with actuarial perceptions of risk. Required capital reflects uncertainty in the unpaid
losses (not any systematic risk or beta of underwriting) and the length of time the reserves are held. If unpaid
losses A and B are equally uncertain, and loss B is held in reserves twice as long as loss A, loss B has about
twice the risk margin as loss A. Similarly, if line B has twice the required capital as line A because its losses
are more risky, line B has twice the risk margin as line A.

The risk margin is an actual cost, not an ephemeral construct. It does not say that loss reserves are risky so
insurers must receive additional reward. It says that insurers incur costs to hold loss reserves. These costs
appear in accounting statements; they can be objectively measured and attributed to specific unpaid losses.
The risk margin is not a return for bearing risk based on theoretical models. It is the actual cost of risk borne
by insurers. The insurer holds a liability for these costs as the risk margin for unpaid losses. As the costs are
paid, the liability reduces.

FAS 157 says that the risk margin depends on the market. If insurers transfer all unpaid losses to Bermuda
based reinsurers with low capital requirements and tax liabilities, Bermuda regulation governs the cost of
holding capital. If insurers retain their unpaid losses, the host country regulations govern the cost of holding
capital. Most unpaid losses are not traded in any active market, so host country regulations govern.

This heuristic illustration shows the pieces needed for the fair value of casualty loss reserves: the present
value of the loss payments, the required capital, the cost of holding capital, and the tax on this cost. Required
capital has two forms: implicit capital in full value loss reserves and explicit capital in surplus. An insurer that
transfers unpaid losses to another party no longer holds supporting capital, which the other party now holds.

COST OF HOLDING CAPITAL

The cost of holding capital connects the target return on capital and the indicated premium rate. A problem
with terminology has plagued many discussions of this topic. To clarify the terms, we differentiate between
the cost of capital and the cost of holding capital.

- The cost of capital is the return demanded by owners or other suppliers of capital. The cost of capital may
  be 8% for long-term debt, 13% for retained earnings, and somewhat higher for a new stock issue. The
cost of equity capital is greater than the cost of debt capital because of the greater systematic risk of
equity.
- The cost of holding capital is what equity-holders would lose by providing capital to the insurer were they
  not compensated by a profit margin in the policy premium.

The cost of capital is observed: it is the average returns to stockholders and bondholders. The cost of holding
capital is inferred from tax law and financial friction costs related to investment of capital. Double taxation is
the primary cost of holding capital. Investors supplying capital to an insurer are taxed twice on the investment
income on capital funds.

Suppose an insurer needs $100 million of capital to support its operations, and the risk-free rate is 10%. The
cost of capital is the return its owners receive if they invest the $100 million in projects of similar risk. The cost
of holding capital is the difference between this cost and the return received by investment through the insurer.

- Owners could invest $100 million in bonds yielding 10%, on which they pay personal income taxes.
- If the insurer makes the same investment, it pays $3.5 million of corporate income taxes before remitting
  the remaining $6.5 million to its owners, who then pay personal income taxes on this return.
The cost of double taxation is the difference in the taxes incurred between (i) direct investment of capital and (ii) investment of capital through an insurer.

- The taxes paid on direct investment of capital = \( \text{investment yield} \times \text{personal tax rate} \).
- The taxes paid on investment of capital through an insurer = \( \text{investment yield} \times \left[ \text{corporate tax rate} + (1 - \text{corporate tax rate}) \times \text{personal tax rate} \right] \).
- The difference is
  \[
  \text{investment yield} \times \left[ \text{corporate tax rate} + (1 - \text{corporate tax rate}) \times \text{personal tax rate} - \text{personal tax rate} \right] = \text{investment yield} \times \text{corporate tax rate} \times (1 - \text{personal tax rate})
  \]

This is the after-tax difference to owners. The difference before personal income taxes is the \( \text{investment yield} \times \text{the corporate tax rate} \).

**Personal Tax Rates**

Miller [1977] suggested a change to this cost of holding capital. The computation above assumes shareholders have the same tax rate on bonds and stocks. If stockholders' tax rates differ for bonds vs stocks, we reconsider the combined effect.²

Personal taxpayers have an average 32% to 36% tax rate on bonds, depending on their tax bracket. The effective tax rate on common stocks depends on the holding period, the pre-tax yield, and the ratio of dividends to capital accumulation. The average insurance stock yields are 12% per annum, half dividends and half capital gains, and investors hold the stocks an average of ten years before realizing gains.

In ten years, one dollar accumulates to $3.106 at a 12% annual rate. The after-tax gain upon realization is \( 85\% \times ($3.106 - $1) = $1.79 \). The after-tax investment yield needed to achieve this gain is \( ($1.79 + $1)^{1/10} - 1 = 10.81\% \), for an effective tax rate of \( 1 - 10.81\% / 12\% = 9.92\% \). Assuming a split of 50% dividends and 50% capital gains, the marginal tax rate on common stocks is \( \frac{1}{2} \times (15\% + 9.92\%) = 12.46\% \).

The effective tax rate on stocks is about 12.5%. The difference from the personal tax rate on interest income is about 21.5%, and the cost of double taxation is about 13%.

**Tax Clienteles**

Miller’s emendation seemed reasonable in 1977, before people understood why insurers held bonds and personal taxpayers held mostly stocks. Miller reasoned that the insurer invested in bonds, so we must compare the after-tax return from the insurer with the after-tax return of people investing directly in bonds. But people do not invest directly in bonds (except to diversify their portfolios), because personal taxpayers have a comparative advantage to invest in common stocks. We must compare scenarios where all parties invest where they have comparative advantages. We compare the after-tax return to the investors in an insurer which invests in bonds, or \( (1 - \tau) \times (1 - 35\%) \), with the after-tax return to investors in common stock, or \( 1 - \tau \), where \( \tau \) is the effective tax rate for personal taxpayers holding common stock. The cost of double taxation is 35%, not 13%.

Financial economists do not all agree on the cost of double taxation. This syllabus reading does not specify which estimate best reflects the cost. But all agree that capital is costly.

**Other costs of holding capital**

Double taxation is not the only cost of holding capital. Even if there were no corporate income taxes, people are reluctant to give their money to others to invest for them. These agency costs and financial friction costs are particularly important in emerging markets.
We distinguish between investment advisors (or investment vehicles) and investment through another party. Most people invest their money on the advice of others or through investment vehicles, such as mutual funds. The investment advisor or mutual fund receives a small fixed fee. The largest U.S. mutual fund (Vanguard) has an average fee of about 0.40%, which is lower than the transaction costs of private investors. These fees are transaction costs and are not relevant to fair values.

Agency costs refer to giving managers the capital to build plants and develop new products at their discretion. The return to investors is the higher value of the firm run by the managers. American business theory assumes that managers serve the interests of the owners of the firm. This is theory: managers are agents of the owners (the principals). But managers serve their own interests. Owners align these interests with their own by incentive compensation schemes and executive options, but rarely are they completely successful.

Acquisitions are a good example of agency costs. Suppose Insurer ABC does extremely well in 20X4, earning $500 million more than expected, and it has no good investment opportunities for this money. Investor would like to receive the money as dividends or stock buy-backs, but this does not benefit the managers of ABC. Instead, the managers may use the money to acquire Insurer XYZ. Some acquisitions raise the value of the firm, especially if they allow entry into new, profitable markets. But many acquisitions lose money for investors. They help managers, by creating larger firms for them to run, with higher salaries and other benefits.

Double taxation is a high cost in the U.S., which taxes both corporate income and the asset income of personal taxpayers. Agency costs are low, because of the full disclosure required by the SEC of publicly traded firms. In other countries, double taxation is less important, either because corporate income tax rates are low or because personal tax rates on asset income are low. In developing countries, agency costs are high, because accounting rules are lax and firms do not disclose much to investors. Because investors do not know how managers are running the firm, they demand higher returns on capital.

Quantifying the cost of holding capital

Some actuaries quantify the cost of holding capital as the cost of capital minus the insurer’s after-tax investment yield (see Atkinson and Dallas [2000]). This perspective over-states the cost if the insurer has significant investment risk. It is a reasonable estimate if the insurer’s investment risk is removed from its cost of capital. The illustration below shows one method to quantify the cost. Not all economists agree with this procedure.

**Illustration**: Suppose the risk-free rate is 6% per annum, the insurer’s investment yield is 7%, its cost of capital is 12%, and its assets to capital ratio is 3:1. If it invested in risk-free assets yielding 6%, its cost of capital should be 12% − 3 × (7% − 6%) = 9%. Its cost of holding capital is 9% − 6% × (1 − 35%) = 5.10%.

Endnotes:
1. An orderly transaction assumes enough market exposure that it is not a forced liquidation or distress sale. The fair value is an exit price: the price to sell the asset or transfer the liability at the measurement date; see SFAS 157, ¶ 7.
2. See Miller [1977; 1988] and DeAngelo and Masulis [1980].
ILLUSTRATION A (Case: No Losses) Summary of Cash Flows

$t = 0$

- **ASSETS** $1,250
- **DTA** $70
- **U/W** $750
- **IRS** $17.5
- **FIN MKTS** $412.5
- **EQHR**

$t = 0.5$

- **ASSETS** $750
- **DTA** $35
- **U/W** $150
- **IRS** $113.75
- **FIN MKTS** $47.2
- **EQHR**

$t = 1.0$

- **ASSETS** $0
- **DTA** $35
- **U/W**
- **IRS** $113.75
- **FIN MKTS** $28.6
- **EQHR** $619.84

Comment Letter No. 46
ILLUSTRATION B (Case: Losses Incurred) Summary of Cash Flows
ILLUSTRATION A (No Losses Incurred)

**DTA Due to Revenue Offset**
- \[= 35\% \times (\text{IRS Basis Revenue} - \text{SAP Basis Revenue})\]
- \[= 35\% \times [(\text{WP} - 80\% \times \text{UEPR}) - (\text{WP} - \Delta \text{UEPR})]\]
- \[= 35\% \times 20\% \times \text{UEPR}\]
- \[= 70\]

**DTA Due to IRS Discounting** = 0

The policyholder pays $1000 premium at \(t = 0\), from which the Insurer pays acquisition expenses of $250. The U/W flow to the Insurer at \(t = 0\) is $1000 - $250 = $750

The IRS taxable income is equal to:
- \[\text{WP} - 80\% \times \Delta \text{UEPR} - \text{expenses} - \text{IRS basis Incurred Losses}\]
- \[= \$1000 - 80\% \times \$1000 - \$250 - \$0\]
- \[= \$750\]

Total FIT is equal to:
- \[\text{FIT} = 35\% \times \text{(Taxable Income)}\]
- \[= 35\% \times 750 = \$70\]

Which represents a $70 tax refund.

At \(t = 0\) there is no investment income.

**Required Assets**
- \[\text{UEPR} + \text{Loss Reserves} + \text{Surplus}\]
- \[= \$1000 + 0 + 25\% \times \$1000\]
- \[= \$1250\]

The Insurer collects $750 from U/W operations and a $17.5 tax debit, and $412.5 from the equityholders for a total of $1180.

These funds are invested at a 4% semi-annual rate of return and generate investment income over the period from \(t = 0\) to \(t = 0.5\)

The Insurer needs total assets of $1250, of which $70 is supported by a deferred tax asset. Of the total $1180 of assets that must be funded, $767.5 is funded via company operations. The remainder:
- \[\text{Required Assets} - \text{DTA} - \text{Company Cash Flows}\]
- \[= \$1250 - 70 - 767.5 = \$412.5\]

must come from investors (equityholders).
ILLUSTRATION A (No Losses Incurred)

**DTA Due to Revenue Offset**
- Amortization of $70 DTA at t=0
  - $0.5 \times $70 = $35

**DTA Due to IRS Discounting**
- $0

With DTA at t=0 equal to $70 this implies a DTA flow of -$35.

The insurer pays general expenses of $150. The U/W flow to the insurer at t=0 is -$150.

The IRS taxable U/W income computed at year end (at t=1) is equal to:
- WP - 80\% \cdot UEPR - expenses - IRS basis
- Incurred Losses
  - $0 - 80\%(-$1000) = $150 - $0
  - $650
- The U/W tax calculated at year end is 35\% \cdot taxable U/W Income
  - 35\% \cdot $650 = $227.5
- The semi-annual payment of this year end U/W tax is
  - 1/2 \cdot year end U/W tax
  - 1/2 \cdot $227.5 = $113.75
- The IRS taxable investment income is $47.2. With a tax rate of 35\% this means a tax of 35\% \cdot $47.2 = $16.52 on investment income.
- The total FIT is thus $113.75 + $16.52 = $130.27

**Required Assets**
- UEPR + Loss Reserves + Surplus
- At time 0.5: UEPR = 1/2 \cdot WP = $500,
  - Loss Reserves = $0,
  - Surplus = 25\% \cdot WP + 15\% \cdot (Loss Reserves) = $250
- Required Assets are
  - $500 + $0 + $250 = $750
- Of these assets, $35 are deferred tax assets that are non-investable, and hence $750 - $35 = $715 is investable.

The starting assets at t=0 are $1250. Over the period from t=0 to t=0.5 company operations have drained $233.07 from these funds. The DTA has diminished the asset fund by a further $35 for a total $268.07 decrease and a fund balance of $981.93. However, the amount of assets required at t=0.5 is only $750. The remainder $981.93 - $750 = $231.93 is "free".

**Equity Flow**
- Starting Required Assets + Company Cash Flow + DTA Flow - Ending Required Assets
- = Asset Flow + Company Cash Flow + DTA Flow
ILLUSTRATION A (No Losses Incurred)

**DTA Due to Revenue Offset**
- = amortization of $70 DTA set up at t=0
- = 0 * $70 = $0
- DTA Due to IRS Discounting = $0

With DTA at t=0.5 equal to $35 this implies a DTA flow of -$35.

**t = 1.0**

- There is no U/W activity at t=1.0
- The IRS taxable U/W income computed at year end (at t=1) is equal to:
  - WP - 80%\(\Delta\)UEPR - expenses - IRS basis
  - Incurred Losses
  - = $0 - 80%(-$1000) - $150 - $0
  - = $650
  - The U/W tax calculated at year end is
  - 35% * taxable U/W Income
  - 35% * $650 = $227.5
  - The semi-annual payment of this year end U/W tax is
  - year end U/W tax - tax paid to date
  - = $227.5 - 113.75 = $113.75
  - The IRS taxable investment income is
  - $47.2 With a tax rate of 35% this means a tax of 35% * $47.2 = $16.52 on investment income.
  - The total FIT is thus
  - $113.75 + $16.52 = $130.27

- The investable assets at t=0.5 are equal to $715 The investment rate of return is 4% semi-annually. The investment income earned over the period is
- = $715 * 4% = $28.6

- The starting assets at t=0.5 a period from t=0.5 to t=1.0 have drained $95.16 from the has diminished the asset fund total $130.16 decrease and the $619.84. The amount of asset $0. The balance of is "free" cash fl

- The Company Cash Flow Flow + DTA Flow - Ending I

- Now that the policy is fully earned as have been incurred, the required i equal to $0.

- The Company Cash Flow for t consists of $28.6 investment inc $123.76 of taxes, for a total $95.16.
Now that the policy is fully earned and no losses have been incurred, the required assets are equal to $0.  

The **Company Cash Flow** for this period consists of $28.6 investment income, minus $123.76 of taxes, for a total $95.16 outflow.  

The starting assets at t=0.5 are $750. Over the period from t=0.5 to t=1.0 company operations have drained $95.16 from these funds. The DTA has diminished the asset fund by a further $35 for a total $130.16 decrease and a fund balance of $619.84. The amount of assets required at t=1.0 is $0. The balance of $619.84 is "free" cash flow.  

**EquityFlow** =  
Starting Required Assets + Company Cash Flow + DTA Flow - Ending Required Assets  
= Asset Flow + Company Cash Flow + DTA Flow
ILLUSTRATION B (Case Losses Incurred)

**DTA Due to Revenue Offset**
- Amortization of $70 DTA at t=0
  - $70 * 1/2 = $35

**DTA Due to IRS Discounting at year end:**
- $35% * [(Held Reserves - IRS Reserves) at t=1.0]
  - [(1 - .86) * $800 - (1 - .88) * $800]
  - $5.60

**DTA Due to IRS Discounting at mid-year:**
- Interpolation between value at t=0 and t=1.0
  - $35 + $2.80 = $37.80

**Total DTA** = DTA Due to Revenue Offset + DTA Due to IRS Discounting
- $35 + $2.80 = $37.80

With DTA at t=0 equal to $70, this implies a DTA flow of...

**The Insurer pays general expenses of $150. The U/W flow to the Insurer at t=0 is -$150.**

**The IRS taxable U/W income computed at year end (at t=1) is equal to:**
- WP - 80% * UEPR - expenses - IRS basis
- Incurred Losses
  = $0 - 80%(-$1000) - $150 - (0.86 * $800) = $38

The U/W tax calculated at year end is:
- 35% * taxable U/W Income
  - 35% * $650 = $227.50

The semi-annual payment of this year end U/W tax is:
- 1/2 * year end U/W tax
  = 1/2 * ($13.30) = $6.65

The IRS taxable investment income is $47.2. With a tax rate of 35%, this means a tax of 35% * $47.2 = $16.52 on investment income.

The **total FIT** is thus:
- $6.65 + $16.52 = $23.17

**Required Assets = UEPR + Loss Reserves + Surplus**
- At time 0.5: UEPR = 1/2 * WP = $500,
  - Loss Reserves = 1/2 * $800,
  - Surplus = 25% (WP) + 15%(Loss Reserves)
  = $250 + $60 = $310

Required Assets are:
- $500 + $400 + $310 = $1,210

Of these assets, $37.8 are deferred tax assets that are non-investable, and $1210 - $37.8 = $1172.20 is investable.

**The Company Cash Flow for this period consists of $47.2 investment income, minus $9.87 of taxes, minus $150 of general expenses. For a total $112.67 outflow.**

ILLUSTRATION B (Case Losses Incurred)

DTA Due to Revenue Offset = $0

DTA Due to IRS Discounting at year end:
=35%\* [(Held Reserves - IRS Reserves) at t=1.0 - (Held Reserves - IRS Reserves) at t=2.0]  
=35%\*[(1-.86)*$800 - (1-.88)*$800]  
=5.60

Total DTA = DTA Due to Revenue Offset + DTA Due to IRS Discounting  
=$0 + $5.60 = $5.60

With DTA at t=0.5 equal to $37.80 this implies a DTA flow of -$32.20

There is no U/W activity at t=1.0

The IRS taxable U/W income computed at year end (at t=1) is equal to:
WP - 80%\*UEPR - expenses - IRS basis
Incurred Losses  
=$0 - 80%(-$1000) - $150 - (0.86*$800) = $38

The U/W tax calculated at year end is 35%\*U/W Income  
35%\*(-$38) = -$13.30

The semi-annual payment of this year end U/W tax is year end U/W tax - paid to date  
=-$13.3-(-$6.65)=-$6.65

The IRS taxable investment income is $46.89 With a tax rate of 35% this means a tax of 35%\*$46.89 = $16.41 on investment income.

The total FIT is thus -$6.65 + $16.41 = $9.76

Required Assets = UEPR + Loss Reserves + Surplus
At time 1.0: UEPR =$0, Loss Reserves = $800, Surplus = 25%($0) + 15%($800) = $120 = $120

So, Required Assets are  
=$800 +$120 + $920

Of these assets, $5.60 are deferred tax assets that are non-investable, and hence $920-$5.60 = $914.40 is investable.

The Company Cash Flow for this period consists of $46.89 investment income, minus $9.76 of taxes, for a total $37.13 inflow.

The starting assets at t=0.5 are $1210. Over the period from t=0.5 to t=1.0 company operations have added $37.13 from these funds. The decrease in DTA has diminished the asset fund by $32.20 for a total $4.93 increase and a fund balance of $1214.93. The amount of assets required at t=1.0 is only $920. The balance of $294.93 is "free" cash flow (equity flow).

EquityFlow = Starting Required Assets + Company Cash Flow + DTA Flow - Ending Required Assets  
=-Asset Flow + Company Cash Flow + DTA Flow
ILLUSTRATION B (Case Losses Incurred)

Required Assets = UEPR + Loss Reserves + Surplus

At time 2.5: UEPR = $0, Loss Reserves = $800, Surplus = 25%($0) + 15%(Loss Reserves) $120 = $120

So, Required Assets are $800 + $120 + $920

Of these assets, $16.80 are deferred tax that are non-investable, and $920 - $16.80 = $903.20 is investable.

The starting assets at t=2.0 are $9

period from t=2.0 to t=2.5 company have added $39.85 to these funds. TI DTA has diminished the asset fund by total $23.05 increase and a fund $943.05. The amount of assets require still $920. The balance of $2 is "free" cash flow (equity)

EquityFlow = Starting Required Assets + Company Cash Flow + DTA Flow - Ending Requi

= Asset Flow + Company Cash Flow

The IRS taxable U/W income computed at year end (at t=1) is equal to:
WP - 80%\*UEPR - expenses - IRS basis

Incurred Losses

= $0 - 80%(0) - $0 - (800 + (0 - 0.88\*$800)) = $-96

The U/W tax calculated at year end is 35%\*taxable U/W income
35\%\*(-$96) = -$33.6

The semi-annual payment of this year end U/W tax is 1/2 \* year end U/W tax = 1/2\*(-$33.6) = -$16.8

The IRS taxable investment income is $35.46. With a tax rate of 35% this means a tax of 35\%\*35.46 = $12.41 on investment income.

The total FIT is thus -$16.8 + $12.41 = -$4.39 (a refund)

The investable assets at t=2.0 are equal to $886.40. The investment rate of return is 4% semi-annually. The investment income earned over the period is $886.4 \* 4\% = $35.46

The Company Cash Flow for this period consists of $35.46 investment income and refund of $4.39 for a total $39.85 increase.

The investable assets at t=2.0 are $886.40. The investment rate of return is 4% semi-annually. The investment income earned over the period is $886.4 \* 4\% = $35.46

The starting assets at t=2.0 are $9

period from t=2.0 to t=2.5 company have added $39.85 to these funds. TI DTA has diminished the asset fund by total $23.05 increase and a fund $943.05. The amount of assets require still $920. The balance of $2 is "free" cash flow (equity)

EquityFlow = Starting Required Assets + Company Cash Flow + DTA Flow - Ending Requi

= Asset Flow + Company Cash Flow

The IRS taxable U/W income computed at year end (at t=1) is equal to:
WP - 80\%\*UEPR - expenses - IRS basis

Incurred Losses

= $0 - 80\%(0) - $0 - (800 + (0 - 0.88\*$800)) = $-96

The U/W tax calculated at year end is 35\%\*taxable U/W income
35\%\*(-$96) = -$33.6

The semi-annual payment of this year end U/W tax is 1/2 \* year end U/W tax = 1/2\*(-$33.6) = -$16.8

The IRS taxable investment income is $35.46. With a tax rate of 35% this means a tax of 35\%\*35.46 = $12.41 on investment income.

The total FIT is thus -$16.8 + $12.41 = -$4.39 (a refund)

The investable assets at t=2.0 are equal to $886.40. The investment rate of return is 4% semi-annually. The investment income earned over the period is $886.4 \* 4\% = $35.46

The starting assets at t=2.0 are $9

period from t=2.0 to t=2.5 company have added $39.85 to these funds. TI DTA has diminished the asset fund by total $23.05 increase and a fund $943.05. The amount of assets require still $920. The balance of $2 is "free" cash flow (equity)

EquityFlow = Starting Required Assets + Company Cash Flow + DTA Flow - Ending Requi

= Asset Flow + Company Cash Flow
ILLUSTRATION B (Case Losses Incurred)

**t = 3.0**

- **ASSETS**
  - $0
  - **DTA**
    - $0
  - $16.8
  - **U/W**
    - $800
  - **INSURER**
    - **IRS**
      - $4.16
      - $36.13
      - $143.48
  - **FIN MKTS**
  - **EQHR**

The starting assets at **t = 2.5** are $920. Over the period from **t = 2.5** to **t = 3.0**, the decrease in DTA has drained $759. As a result, the increase in **funds** is $16.80 for a total $759. The balance of $920 consists of $36.13 investment income, plus a taxable **U/W** income of $800, and a fund balance of $903.20.

The **semi-annual payment** of this year end U/W tax is $33.60. The **investable assets at t = 2.5** are equal to $903.20. The **investment income** earned over the period is $36.13. The total FIT is thus $16.80 + $12.64 = $4.16 (a refund).

The **investable assets at t = 2.5** are $903.20. The **investment income** earned over the period is $36.13. The **total FIT** is thus $16.80 + $12.64 = $4.16 (a refund).

The **investable assets at t = 2.5** are $903.20. The **investment income** earned over the period is $36.13. The **total FIT** is thus $16.80 + $12.64 = $4.16 (a refund).
ILLUSTRATION B (Case Losses Incurred)

Loses have been paid, the required assets are equal to $0.

Company Cash Flow for this period of $36.13 investment income, plus a tax of $9.76 of taxes, minus $800 of paid out $759.71 outflow.

Ending assets at t=2.5 are $920. Over the from t=2.5 to t=3.0 company operations drained $759.71 from these funds. The in DTA has diminished the asset fund by for a total $776.51 decrease and a fund of $143.48, assets are required at t=3.0.

The balance of $143.48 is "free" cash flow (equity flow).

# ILLUSTRATION A: Case No Losses

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**IRR on Equityflows** 136.8%

**ASSUMPTIONS**  
- **Invest Rate of Return** (bond equivalent yield) 8.0%  
- **Semi-annual Rate** 4.0%  
- **Losses** 0  
- **Target Return on Capita** 12.0%  
- **Semi-annual Rate** 5.8%
ILLUSTRATION B: Case Losses Incurred (IRR < Target ROC)

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OTHER

DTA

| Revenue Offset | 70.00 | 35.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Due to Loss Reserve Discounting | 0.00 | 2.80 | 5.60 | 19.60 | 33.60 | 16.80 | 0.00 |
| TOTAL | 70.00 | 37.80 | 5.60 | 19.60 | 33.60 | 16.80 | 0.00 |

GAAP ACCOUNTS

| DPAC | 250.00 | 125.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Revenue Offset | -17.50 | -8.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Due to Loss Reserve Discounting | 0.00 | 19.60 | 39.20 | 36.40 | 33.60 | 16.80 | 0.00 |
| TOTAL GAAP DTA | -17.50 | 10.85 | 39.20 | 36.40 | 33.60 | 16.80 | 0.00 |

RESERVES

| UEPR | 1,000.00 | 500.00 | 0.00 |
| Stat Loss Reserve | 400.00 | 800.00 | 800.00 | 800.00 | 800.00 | 800.00 | 800.00 |
| IRS Discount Factors | 0.86 | 0.88 | 0.90 |
| Tax Basis Loss Reserve | 688.00 | 704.00 | 0.00 |
| Surplus | 250.00 | 310.00 | 120.00 | 120.00 | 120.00 | 120.00 | 120.00 |
| Total Assets | 1,250.00 | 1,210.00 | 920.00 | 920.00 | 920.00 | 920.00 | 920.00 |
| Investable Assets | 1,180.00 | 1,172.20 | 914.40 | 900.40 | 886.40 | 903.20 | 0.00 |

TAXES

| Tax Basis UW Revenue | 200.00 | 800.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Tax Basis Expenses | 250.00 | 150.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Tax Basis Inc Loss | 0.00 | 688.00 | 16.00 | 96.00 |
| Tax Basis Inv Income | 0.00 | 47.20 | 46.89 | 36.58 | 36.02 | 35.46 | 36.13 |

Valuation of Taxes

| Tax on UW Income (Annual) | -17.50 | -13.30 | -5.60 | -33.60 |
| Tax on UW Income (Semi-Annual Payment) | -17.50 | -6.65 | -6.65 | -2.80 | -2.80 | -16.80 | -16.80 |
| Tax on Invest Income | 0.00 | 16.52 | 16.41 | 12.80 | 12.61 | 12.41 | 12.64 |
| Total Semi-Annual Payment | -17.50 | 9.87 | 9.76 | 10.00 | 9.81 | 4.39 | 4.16 |

CASH FLOWS

| Asset Flow | 1,250.00 | -40.00 | -290.00 | 0.00 | 0.00 | 0.00 | -920.00 |
| UW Flow | 750.00 | -150.00 | 0.00 | 0.00 | 0.00 | 0.00 | -800.00 |
| Inv Inc Flow | 47.20 | 46.89 | 36.58 | 36.02 | 35.46 | 36.13 |
| Tax Flow | 17.50 | -9.87 | -9.76 | -10.00 | -9.81 | 4.39 | 4.16 |
| DTA Flow | 70.00 | -32.20 | -32.20 | 14.00 | 14.00 | -16.80 | -16.80 |
| Equityflow | -412.50 | -104.87 | 294.93 | 40.57 | 40.21 | 23.05 | 143.48 |

CAPITAL

| EFC | 412.50 | 523.50 | 236.34 | 199.28 | 162.02 | 141.38 | 0.00 |
| Contributed Capital | 412.50 | 111.00 | -287.15 | -37.07 | -37.25 | -20.64 | -141.38 |
| Net Income | 0.00 | 6.13 | 7.77 | 3.51 | 2.96 | 2.64 | 2.10 |
| Value Added | -17.92 | -22.75 | -10.27 | -8.66 | -7.04 | -6.14 |

IRR on Equityflows: (annual rate) 3.0%
(semi-annual rate) 1.5%

ASSUMPTIONS

| Invest Rate of Return | 8.0% | Semi-annual Rate | 4.0% |
| (bond equivalent yield) | | | |
| Losses | $800 | Premium | 1,000.00 |
| Combined Ratio | 120.0% | Loss Ratio | 80.0% |
| Target Return on Capital | 12.0% | Semi-annual Rate | 5.8% |
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### U/W Assumptions
- Target Return on Capital = 12.0%
- Invest Rate of Return = 8.0%
- Premium = $1,000
- Dollars of Ultimate Loss = $0

### Surplus Assumptions
- Premium Leverage Ratio = 25%
- Reserve Leverage Ratio = 15%

### Results
- IRR on Equityflows (annual rate) = 136.8%
## U/W Cash Flows

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<td>0.00</td>
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</table>
### Asset Flows

#### Table of Asset Flows:

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<thead>
<tr>
<th></th>
<th>t = 0</th>
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<tbody>
<tr>
<td>DTA Flow</td>
<td>70.00</td>
<td>-32.20</td>
<td>-32.20</td>
<td>14.00</td>
<td>14.00</td>
<td>-16.80</td>
<td>-16.80</td>
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<tr>
<td>Funded Asset Flow</td>
<td>1,180.00</td>
<td>-7.80</td>
<td>-257.80</td>
<td>-14.00</td>
<td>-14.00</td>
<td>16.80</td>
<td>-903.20</td>
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<tr>
<td>Needed Asset Flow</td>
<td>1,250.00</td>
<td>-40.00</td>
<td>-290.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-920.00</td>
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</table>
### Company Cash Flows

<table>
<thead>
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<tr>
<td>U/W Flow</td>
<td>750.00</td>
<td>-150.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-800.00</td>
</tr>
<tr>
<td>Invest Income</td>
<td>0.00</td>
<td>47.20</td>
<td>46.89</td>
<td>36.58</td>
<td>36.02</td>
<td>35.46</td>
<td>36.13</td>
</tr>
<tr>
<td>Tax Flow</td>
<td>17.50</td>
<td>-9.87</td>
<td>-9.76</td>
<td>-10.00</td>
<td>-9.81</td>
<td>4.39</td>
<td>4.16</td>
</tr>
<tr>
<td>Co Cash Flow</td>
<td>767.50</td>
<td>-112.67</td>
<td>37.13</td>
<td>26.57</td>
<td>26.21</td>
<td>39.85</td>
<td>-759.72</td>
</tr>
</tbody>
</table>
Equity Flows

Funded Asset Flow
-1,500.00  -1,000.00  -500.00  0.00  500.00  1,000.00  1,500.00  
Company Cash Flow
767.50  -112.67  37.13  26.57  26.21  39.85  -759.72  
Equity Flow
-412.50  -104.87  294.93  40.57  40.21  23.05  143.48  

- t = 0  t = 0.5  t = 1.0  t = 1.5  t = 2.0  t = 2.5  t = 3.0 -
### TAX DECOMPOSITION

<table>
<thead>
<tr>
<th>Tax Basis</th>
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<tbody>
<tr>
<td>(1) UW Revenue</td>
<td>200.00</td>
<td>800.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(2) Expenses</td>
<td>250.00</td>
<td>150.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(3) Inc Loss</td>
<td>0.00</td>
<td>688.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>96.00</td>
</tr>
<tr>
<td>(4) Inv Income</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(4a) on funds due to (Rev - Exp)</td>
<td>0.00</td>
<td>47.20</td>
<td>28.60</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(4b) on funds due to Incurred of L</td>
<td>0.00</td>
<td>0.00</td>
<td>18.29</td>
<td>36.58</td>
<td>36.02</td>
<td>35.46</td>
<td>36.13</td>
</tr>
<tr>
<td>(4c) Total Inv Income</td>
<td>0.00</td>
<td>47.20</td>
<td>46.89</td>
<td>36.58</td>
<td>36.02</td>
<td>35.46</td>
<td>36.13</td>
</tr>
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</table>

<table>
<thead>
<tr>
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<th>t = 2.5</th>
<th>t = 3.0</th>
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</thead>
<tbody>
<tr>
<td>(5) U/W Revenue - Expenses</td>
<td>-17.50</td>
<td>227.50</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(6) Inc Loss</td>
<td>0.00</td>
<td>-240.80</td>
<td>-5.60</td>
<td>-33.60</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7) Inv Inc</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7a) on funds due to (Rev - Exp)</td>
<td>0.00</td>
<td>16.52</td>
<td>10.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(7b) on funds due to Incurred of L</td>
<td>0.00</td>
<td>0.00</td>
<td>6.40</td>
<td>12.80</td>
<td>12.61</td>
<td>12.41</td>
<td>12.64</td>
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<table>
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<th>t = 1.5</th>
<th>t = 2.0</th>
<th>t = 2.5</th>
<th>t = 3.0</th>
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</thead>
<tbody>
<tr>
<td>(8) Writing of Policy (semi-ann payment)</td>
<td>-17.50</td>
<td>130.27</td>
<td>123.76</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(9) Incurred of Loss (semi-ann payment)</td>
<td>-120.40</td>
<td>-114.00</td>
<td>10.00</td>
<td>9.81</td>
<td>-4.39</td>
<td>-4.16</td>
<td></td>
</tr>
<tr>
<td>(10) Total Tax (semi-ann payment)</td>
<td>-17.50</td>
<td>9.87</td>
<td>9.76</td>
<td>10.00</td>
<td>9.81</td>
<td>-4.39</td>
<td>-4.16</td>
</tr>
</tbody>
</table>

### FORMULAS

- (5) = 0.35 * [(1) - (2)]
- (6) = -0.35 * (3)
- (7a) = 0.35 * (4a)
- (7b) = 0.35 * (4b)
- (8) t = (7a) t + 0.5 * [ (5) t+0.5 ], for t = 0.5, 1.5, 2.5
- (9) t = (7b) t + 0.5 * [ (6) t+0.5 ], for t = 0.5, 1.5, 2.5
- (10) t = (7a) t + (7b) t + 0.5 * [ (5) t+0.5 + (6) t+0.5 ], for t = 0.5, 1.5, 2.5
### Deferred Tax Asset

#### (1) Tax Basis

<table>
<thead>
<tr>
<th></th>
<th>( t = 0 )</th>
<th>( t = 1.0 )</th>
<th>( t = 2.0 )</th>
<th>( t = 3.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Revenue</td>
<td>200.00</td>
<td>800.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>b. Expense</td>
<td>250.00</td>
<td>150.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>c. Incurred Loss</td>
<td>0.00</td>
<td>688.00</td>
<td>16.00</td>
<td>96.00</td>
</tr>
<tr>
<td>d. Tax due to Revenue</td>
<td>70.00</td>
<td>280.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>e. Tax due to Expense</td>
<td>-87.50</td>
<td>-52.50</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>f. Tax due to Losses</td>
<td>0.00</td>
<td>-240.80</td>
<td>-5.60</td>
<td>-33.60</td>
</tr>
<tr>
<td>g. Tax on U/W Total</td>
<td>-17.50</td>
<td>-13.30</td>
<td>-5.60</td>
<td>-33.60</td>
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#### (2) Statutory Basis

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<th>( t = 1.0 )</th>
<th>( t = 2.0 )</th>
<th>( t = 3.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Revenue</td>
<td>0.00</td>
<td>1,000.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>b. Expense</td>
<td>250.00</td>
<td>150.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>c. Incurred Loss</td>
<td>0.00</td>
<td>800.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>d. Tax due to Revenue</td>
<td>0.00</td>
<td>350.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>e. Tax due to Expense</td>
<td>-87.50</td>
<td>-52.50</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>f. Tax due to Losses</td>
<td>0.00</td>
<td>-280.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>g. Tax on U/W Total</td>
<td>-87.50</td>
<td>17.50</td>
<td>0.00</td>
<td>0.00</td>
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#### (3) GAAP Basis

<table>
<thead>
<tr>
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<th>( t = 1.0 )</th>
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<th>( t = 3.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Revenue</td>
<td>0.00</td>
<td>1,000.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>b. Expense</td>
<td>250.00</td>
<td>150.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>b' DPAC</td>
<td>250.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>c. Incurred Loss</td>
<td>0.00</td>
<td>800.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>d. Tax due to Revenue</td>
<td>0.00</td>
<td>350.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>e. Tax due to (Expense - ( \Delta DPAC))</td>
<td>0.00</td>
<td>-140.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>f. Tax due to Losses</td>
<td>0.00</td>
<td>-280.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>g. Tax on U/W Total</td>
<td>0.00</td>
<td>-70.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

#### FORMULAE

\[
(4a)_t = (2d)_t - (1d)_t \\
(4b)_t = (2f)_t - (1f)_t \\
(4c) = (4a) + (4b) \\
(4d)_{t+1} = (4d)_t + (4c)_{t+1} \\
(4e)_t = (4d)_{t+1} - (4b)_{t+1} 	ext{ for } t \geq 0
\]

#### (4) Statutory DTA Flow

<table>
<thead>
<tr>
<th></th>
<th>( t = 0 )</th>
<th>( t = 1.0 )</th>
<th>( t = 2.0 )</th>
<th>( t = 3.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. due to Revenue Offset</td>
<td>70.00</td>
<td>-70.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>b. due to Loss Reserve Discounting</td>
<td>0</td>
<td>39.20</td>
<td>-5.6</td>
<td>-33.6</td>
</tr>
<tr>
<td>c. DTA Flow w/out Reversal</td>
<td>70.00</td>
<td>-30.80</td>
<td>-5.60</td>
<td>-33.60</td>
</tr>
<tr>
<td>d. DTA w/out Reversal</td>
<td>70.00</td>
<td>39.20</td>
<td>33.60</td>
<td>0.00</td>
</tr>
<tr>
<td>e. DTA w/ Reversal</td>
<td>70.00</td>
<td>5.60</td>
<td>33.60</td>
<td>0.00</td>
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#### (5) GAAP DTA Flow

<table>
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<th>( t = 3.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. due to Revenue Offset</td>
<td>-17.50</td>
<td>56.70</td>
<td>-5.60</td>
<td>-33.60</td>
</tr>
<tr>
<td>b. due to Loss Reserve Discounting</td>
<td>-17.50</td>
<td>39.20</td>
<td>33.60</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Statutory DTA w/ Reversal is that portion of the DTA that reverses in the year. Hence, it is by definition the negative of the DTA flow w/out reversal that occurs in the subsequent period.
Determination of the Equity Flow

Composition of Assets at time \( t \) is a function of business environment constraints:

\[
\text{Assets}_t = \text{UEPR}_t + \text{Loss Reserve}_t + \text{Surplus}_t
\]

Cash Flow Definition of Equity Flow:

\[
\text{Equity Flow}_t = -\text{Asset Flow}_t + \text{Company Cash Flow}_t
\]

Income Statement Definition of Equity Flow:

\[
\text{Equity Flow}_t = \text{Accting Net Income}_t - \Delta (\text{Accting Capital})_t
\]

For the above Illustration:

\[
\text{Asset Flow}_t = (4-7) = -3
\]

\[
\text{Company Cash Flow}_t = \text{Co Cash Inflow}_t - \text{Co Cash Outflow}_t
\]

\[
= +2 - 3 = -1
\]

\[
\therefore \text{Equity Flow}_t = -(-3) + (-1) = 2
\]
Cash Flow View of the Equity Flow

<table>
<thead>
<tr>
<th>Required Assets at t=0</th>
<th>Cash Flow Activity from t=0 to t=0.5</th>
<th>Accumulated Assets at t=0.5</th>
<th>Required Assets at t=0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surplus $250</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UEPR $1,000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Equity Flow (inflow to EQHRs)

= Accumulated Assets - Ending REQ Assets
  = $981.93 - $750
  = $231.93

Company Cash Inflow

Investment Income: $47.20

Company Cash Outflow

expenses: $150
taxes: $130.27

Passage of time

= Starting REQ Assets + Company Cash Inflow - Company Cash Outflow - DTA Outflow
  =$1,250 + $47.20 - $280.27 - $35
  =$981.93
# Income Statement View of Equity Flows

<table>
<thead>
<tr>
<th>STATEMENT OF INCOME</th>
<th>0.00</th>
<th>0.50</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earned Premium</td>
<td>500</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>Undiscounted Incurred Loss</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Paid Taxes</td>
<td>130.27</td>
<td>123.76</td>
<td></td>
</tr>
<tr>
<td>Undiscounted Incurred Expenses</td>
<td>150</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Incurred Costs</td>
<td>280.27</td>
<td>123.76</td>
<td></td>
</tr>
<tr>
<td>Investment Income</td>
<td>47.2</td>
<td>28.6</td>
<td></td>
</tr>
<tr>
<td>Change in DTA</td>
<td>-35</td>
<td>-35</td>
<td></td>
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<tr>
<td>Net Income</td>
<td>231.93</td>
<td>369.84</td>
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</table>

<table>
<thead>
<tr>
<th>CAPITAL and SURPLUS ACCOUNT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surplus, end of previous period</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GAINS (and LOSSES) IN SURPLUS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Income</td>
<td>231.93</td>
</tr>
<tr>
<td>Surplus Adjustments (Capital Contributions)</td>
<td>0</td>
</tr>
<tr>
<td>Surplus, end of current quarter</td>
<td>250</td>
</tr>
<tr>
<td>Implied Equityflow</td>
<td>231.93</td>
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</table>